The Arithmetic Teacher

DEC 2 1959

NOVEMBER • 1959

Similarities Between Language and Arithmetic
J. ALLEN HICKERSON

Providing for the Gifted Child
ALICE PANEK

"Plus" Work for "Plus" Pupils
EUNICE LEWIS AND ROBERT PLATH

A Gifted Underachiever in Arithmetic
ANN F. ISAACS

Blocks to Arithmetical Understanding
CALHOUN C. COLLIER

A Journal of
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE ARITHMETIC TEACHER

a journal of
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editors: MARGUERITE BRYDEGAARD, San Diego State College, San Diego, Calif.; E. GLENADINE GIBB, State Teachers College, Cedar Falls, Iowa; JOHN R. CLARK, New Hope, Pa.; JOSEPH J. URBANCEK, Chicago Teachers College, Chicago 21, Ill.

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth St., N.W., Washington 6, D. C.

Second class postage paid at Washington, D. C., and at additional mailing offices.

OFFICERS OF THE NATIONAL COUNCIL:

President: Harold P. Fawcett, Columbus, Ohio
Executive Secretary: M. H. Ahrendt, Washington, D. C.
Past President: Howard F. Fehr, New York, N.Y.
Vice Presidents: Ida Bernhard Pruett, Atlanta, Ga.; E. Glenadine Gibb, Cedar Falls, Iowa; Phillip S. Jones, Ann Arbor, Mich; Mildred B. Cole, Aurora, Ill.

DIRECTORS: Clifford Bell, Los Angeles, Calif.; Anne J. Williams, Raleigh, N.C.; Robert E. K. Rourke, Kent, Conn.; Philip Peak, Bloomington, Ind.; Frank B. Allen, La Grange, Ill.; Burton W. Jones, Boulder, Colo.; Bruce E. Meserve, Montclair, N.J.; Oscar F. Schaaf, Eugene, Ore.; H. Van Engen, Madison, Wis.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council (For an additional $3.00 ($1.00 to students) the subscriber may also receive THE MATHEMATICS TEACHER.) Institutional Subscription: $7.00 per year. Single copies: 85¢ each. Remittance should be made payable to the National Council of Teachers of Mathematics. 1201 Sixteenth St., N. W., Washington 6, D. C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

Table of Contents

		Page
Similarities Between Teaching Language and Arithmetic	*J. Allen Hickerson*	241
Report of the Policy Conference of the National Council		244
Providing for the Gifted Child	*Alice Panek*	246
"Plus" Work for "Plus" Pupils	*Eunice Lewis and Ernest C. Plath*	251
A Gifted Underachiever in Arithmetic	*Ann F. Isaacs*	257
Blocks to Arithmetical Understanding	*Calhoun C. Collier*	262
Building Algebra Readiness in Grades Seven and Eight	*Francis J. Mueller*	269
Articulation in Arithmetic	*Peter A. Stearn*	273
Prime Numbers and Factoring	*Frederick L. Goodman*	274
Stocks as Investments	*Thomas J. Ganley*	276
An Adventure in Topology—Grade 5	*Jean C. Clancy*	278
Uses of Styrofoam in Arithmetic	*Adrien L. Hess and Lillian Kearby*	280
Reviews		281
Operating Budget of the National Council of Teachers of Mathematics		284
Subtraction from Left to Right	*David Erlich*	286

THE ARITHMETIC TEACHER

Volume VI Number 5

November 1959

Similarities Between Teaching Language and Arithmetic

J. Allen Hickerson*

New Haven State Teachers College, Conn.

During the last 30 years we have discovered a great deal about how children learn language communication. As a consequence, instructional programs in oral and written expression, listening, reading, and grammar have improved considerably in recent years.

Since arithmetic is a system of symbolisms, just as language is a system of symbolisms, why shouldn't the accepted principles underlying the understanding and use of *language* symbols apply also to the understanding and use of *arithmetic* symbols? It is the writer's conviction that they should apply.

There is evidence to indicate that arithmetic instruction and learning are improved considerably when, what the writer calls, THE EXPERIENCE-LANGUAGE APPROACH TO NUMBERS** is followed. This approach to arithmetic is based on what are considered to be the best ways of teaching a language—whether it is the native language or a foreign language.

The following is an attempt to show point-by-point similarities between good language instruction and good arithmetic instruction. Just how good this approach is to arithmetic teaching requires further experimentation and testing.

I

Similarities between Language and Arithmetic

Language	Arithmetic
1) Language symbols (words or sentences) represent things, actions, ideas, relationships, etc.	1) Arithmetic symbols (numerals and numbers with operational signs) represent things, actions, ideas, relationships, etc.
2) The meaning of language symbols derive from that which they represent.	2) The meaning of arithmetic symbols derive from that which they represent.

* Many teachers who have worked with Dr. Hickerson and who have heard him lecture at various meetings will be saddened to learn of his untimely death early in the summer.

** The Experience-Language Approach to Numbers is discussed more fully by the author in his *Guiding Children's Arithmetic Experiences*, published by Prentice-Hall, Inc. in 1952.

THE ARITHMETIC TEACHER

a journal of
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editors: MARGUERITE BRYDEGAARD, San Diego State College, San Diego, Calif.; E. GLENADINE GIBB, State Teachers College, Cedar Falls, Iowa; JOHN R. CLARK, New Hope, Pa.; JOSEPH J. URBANCEK, Chicago Teachers College, Chicago 21, Ill.

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth St., N.W., Washington 6, D. C.

Second class postage paid at Washington, D. C., and at additional mailing offices.

OFFICERS OF THE NATIONAL COUNCIL:

President: Harold P. Fawcett, Columbus, Ohio
Executive Secretary: M. H. Ahrendt, Washington, D. C.
Past President: Howard F. Fehr, New York, N.Y.
Vice Presidents: Ida Bernhard Pruett, Atlanta, Ga.; E. Glenadine Gibb, Cedar Falls, Iowa; Phillip S. Jones, Ann Arbor, Mich; Mildred B. Cole, Aurora, Ill.

DIRECTORS: Clifford Bell, Los Angeles, Calif.; Anne J. Williams, Raleigh, N.C.; Robert E. K. Rourke, Kent, Conn.; Philip Peak, Bloomington, Ind.; Frank B. Allen, La Grange, Ill.; Burton W. Jones, Boulder, Colo.; Bruce E. Meserve, Montclair, N.J.; Oscar F. Schaaf, Eugene, Ore.; H. Van Engen, Madison, Wis.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.50 to students) includes membership in the Council (For an additional $3.00 ($1.00 to students) the subscriber may also receive THE MATHEMATICS TEACHER.) Institutional Subscription: $7.00 per year. Single copies: 85¢ each. Remittance should be made payable to the National Council of Teachers of Mathematics. 1201 Sixteenth St., N. W., Washington 6, D. C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

Table of Contents

		Page
Similarities Between Teaching Language and Arithmetic	*J. Allen Hickerson*	241
Report of the Policy Conference of the National Council		244
Providing for the Gifted Child	*Alice Panek*	246
"Plus" Work for "Plus" Pupils	*Eunice Lewis and Ernest C. Plath*	251
A Gifted Underachiever in Arithmetic	*Ann F. Isaacs*	257
Blocks to Arithmetical Understanding	*Calhoun C. Collier*	262
Building Algebra Readiness in Grades Seven and Eight	*Francis J. Mueller*	269
Articulation in Arithmetic	*Peter A. Stearn*	273
Prime Numbers and Factoring	*Frederick L. Goodman*	274
Stocks as Investments	*Thomas J. Ganley*	276
An Adventure in Topology—Grade 5	*Jean C. Clancy*	278
Uses of Styrofoam in Arithmetic	*Adrien L. Hess and Lillian Kearby*	280
Reviews		281
Operating Budget of the National Council of Teachers of Mathematics		284
Subtraction from Left to Right	*David Erlich*	286

THE ARITHMETIC TEACHER

Volume VI Number 5

November 1959

Similarities Between Teaching Language and Arithmetic

J. Allen Hickerson*

New Haven State Teachers College, Conn.

During the last 30 years we have discovered a great deal about how children learn language communication. As a consequence, instructional programs in oral and written expression, listening, reading, and grammar have improved considerably in recent years.

Since arithmetic is a system of symbolisms, just as language is a system of symbolisms, why shouldn't the accepted principles underlying the understanding and use of *language* symbols apply also to the understanding and use of *arithmetic* symbols? It is the writer's conviction that they should apply.

There is evidence to indicate that arithmetic instruction and learning are improved considerably when, what the writer calls, THE EXPERIENCE-LANGUAGE APPROACH TO NUMBERS** is followed. This approach to arithmetic is based on what are considered to be the best ways of teaching a language—whether it is the native language or a foreign language.

The following is an attempt to show point-by-point similarities between good language instruction and good arithmetic instruction. Just how good this approach is to arithmetic teaching requires further experimentation and testing.

I

Similarities between Language and Arithmetic

Language	Arithmetic
1) Language symbols (words or sentences) represent things, actions, ideas, relationships, etc.	1) Arithmetic symbols (numerals and numbers with operational signs) represent things, actions, ideas, relationships, etc.
2) The meaning of language symbols derive from that which they represent.	2) The meaning of arithmetic symbols derive from that which they represent.

* Many teachers who have worked with Dr. Hickerson and who have heard him lecture at various meetings will be saddened to learn of his untimely death early in the summer.

** The Experience-Language Approach to Numbers is discussed more fully by the author in his *Guiding Children's Arithmetic Experiences*, published by Prentice-Hall, Inc. in 1952.

3) Communication through language involves speaking, listening, writing, or reading.
4) The study of language is separated into semantics, mechanics, and grammar. (a) *Semantics:* the origin and meanings of words, i.e., what word symbols represent; (b) *mechanics* of manipulating and recognizing language symbols: pronunciation, enunciation, inflection (speaking); hearing and distinguishing spoken sounds (listening); penmanship, spelling, sentence structure, punctuation (writing); eye movements, letter and word recognition, word analysis, eye span (silent reading); pronunciation, enunciation, eye span, expression (oral reading); (c) *grammar:* etymology (the nature of words, e.g., parts of speech) and syntax (the relationship of words in sentences); the rules, principles, generalizations concerning the nature of the structure of the language symbolism.

3) Communication through arithmetic involves speaking, listening, writing, or reading.
4) The study of arithmetic is separated into (a) the *meanings* of arithmetical symbols: what the symbols represent, i.e., the quantities of things and quantitative relations among things; (b) *mechanics* of manipulating and recognizing arithmetic symbols; pronunciation and enunciation of the vocabulary of arithmetic (speaking); hearing and distinguishing sounds of spoken arithmetic words (listening); formation of numbers, signs, and symbols, and structure of algorisms (writing); eye movements, recognition of single numerals and multidigit numbers, recognition of signs and symbols, eye span (reading); (c) *the nature of the structure of the number system*, i.e., number notation and the rules, principles, generalizations concerning the nature of the arithmetic symbolism.

II

Similarities between Learning Language and Learning Arithmetic

LEARNING LANGUAGE	LEARNING ARITHMETIC
1) Engaging in first-hand multi-sensory experiences. (Learning to develop a perceptual content of the biological and physical worlds.)	1) Engaging in first-hand multi-sensory experiences. (Learning to develop a perceptual content of the quantitative aspects and quantitative problem-situations of the biological and physical worlds.)
2) Listening to spoken word-symbols, singly and in sentences, which represent the things, ideas, and events experienced. (Learning vocabulary and sentence structure.)	2) Listening to spoken word-symbols, singly and in sentences, which represent the quantitative aspects, quantitative relationships, or quantitative problem-situations found in the things and events experienced. (Learning the vocabulary and sentence structure used in describing things and what is happening to things.)
3) Representing things, ideas, and events through oral language symbols. (Learning to express self and relate experiences orally.)	3) Representing quantitative aspects, relationships, and problem-situations orally. (Learning to express orally in sentences the quantitative situation, learning to compute orally, and to solve problem situations orally.)

4) Identifying written language-symbols and relating them to spoken language-symbols and to first-hand experiences. (Learning to read with meaning.)

5) Representing things, ideas, and events with written language-symbols. (Learning to express self and relate experiences in writing.)

6) Acquiring knowledge and understanding of the logic of the language structure. (Learning grammatical definitions, rules, principles, generalizations; e.g., parts of speech and syntax.)

4) Identifying written arithmetic-symbols and relating them to spoken word-symbols and to first-hand quantitative experiences. (Learning to read arithmetical symbols with meaning.)

5) Representing quantitative aspects, relationships, and problem situations with written arithmetic-symbols. (Learning to write numbers and operational signs in arithmetic expressions which represent quantitative situations and learning to write algorisms in computation.)

6) Acquiring knowledge and understanding of the logic of the number system. (Learning mathematical definitions, rules, principles, generalizations; e.g., notation, place value, laws of association, distribution, and commutation.)

III

Implications for Teaching

LANGUAGE IS LEARNED BEST WHEN

1) the learner engages in many varied first-hand multi-sensory experiences;

2) oral vocabulary and sentence structure are acquired in relation to the learner's experiences by listening to and talking about the things experienced;

3) written words are read as symbols standing for already known spoken words;

4) the written or spoken symbols have meaning to the reader or listener when they represent something perceived in his experience;

5) the writing of words and sentences is learned after the learner can read;

6) language usage is acquired in childhood gradually by imitation, experimentation, correction—not by memorizing grammatical rules and applying them.

7) grammatical rules, principles, and generalizations are taught by the inductive-deductive method, i.e., the learner is helped to educe and formulate rules,

ARITHMETIC IS LEARNED BEST WHEN

1) the learner engages in many varied first-hand multi-sensory quantitative problem-situations;

2) oral language is acquired which represents in complete sentence form the quantitative relations in problem situations;

3) written arithmetic symbols are introduced as shorthand ways of writing already known spoken words;

4) the written or spoken arithmetic symbols have meaning to the reader or listener when they represent something perceived in his experience;

5) the writing of numbers, number combinations, algorisms, etc., is learned after the learner can read them;

6) computational processes are acquired gradually by manipulation of objects, imitation, experimentation, discovery, correction—not by memorizing mathematical rules and applying them.

7) mathematical rules, principles, and generalizations are taught by the inductive-deductive method, i.e., the learner is helped to educe and formulate rules,

principles, and generalizations from past experience and then test them in subsequent experience.

8) there is a continuous interrelationship existing among widening and deepening first-hand experiences, oral communication, written communication, and increasing consciousness and knowledge of the nature of language.

principles, and generalizations from past experience and then test them in subsequent experience.

8) there is a continuous interrelationship existing among widening and deepening first-hand experiences with the quantitative aspects of life, expression of these experiences with oral and written arithmetical symbolism, and increasing consciousness and knowledge of the nature of arithmetic.

EDITOR'S NOTE. While some of the symbolism of arithmetic may be much more compact than the printed words of language, both of these are none the less symbols. A child must learn the ideas and concepts they express. And parrot-like reproduction of sounds certainly does not indicate understanding of either language or arithmetic. Arithmetic becomes a little more abstract and certain symbols indicate relationships that are not matched in language even though these symbols have names which can be used in writing. For example, $\div$, $\sqrt{}$, %, π, and χ^2. But the pattern of learning might be very much the same and we are very appreciative of the late professor Hickerson for having prepared this manuscript. His insight into the problems youngsters have in learning to understand language and arithmetic plus their abilities to think correctly in these media should encourage all of us to continue to gain a greater insight into the problems of teaching.

Report of the Policy Conference of the National Council of Teachers of Mathematics

CONRAD HILTON HOTEL, CHICAGO
OCTOBER 9–10, 1959

ABOUT thirty leaders in mathematics teaching met in Chicago, October 9th and 10th, 1959. The main purpose of this meeting was to consider the role of The National Council of Teachers of Mathematics in the changing picture of mathematics education at the present and for the future. A report of this conference will be published in the near future.

President Fawcett opened the conference by making the following statement:

"The National Council of Teachers of Mathematics was established in February, 1920, its stated purpose being 'to assist in promoting the interest of mathematics in America, especially in the elementary and secondary fields.' Since its creation nearly 40 years ago it has been guided by this purpose and during this period its services to both elementary and secondary teachers of mathetmatics have steadily increased. Within the last decade or so, developments on a world front have provided a long overdue awakening to the great importance of mathematics in the educational program of our young people and today the improvement of mathematics education from kindergarten through college is a problem which has commanded the active interest of many professional organizations and is of concern to all thoughtful people. Significant curricular studies are in process. New instructional materials are being developed. An in-service

program of large proportions is now operating and the collegiate program for the mathematics teachers of tomorrow is under revision. Large financial assistance is available for all of these projects and it is evident that the original purpose of the Council is now receiving wide support. What then should be the role of the Council in this changed intellectual climate? The School Mathematics Study Group, for example, is developing curricular materials for grades 7 through 12. On the Illinois campus studies are in process which involve the mathematics curriculum on both the elementary and secondary levels. The Council has an Elementary School Curriculum Committee and a Secondary School Curriculum Committee. What should be the relation of these committees to the work of these groups? How can we most effectively cooperate with all such groups in promoting the interests of mathematics in America?

"Mathematics teachers throughout the country are awakening to the important fact that mathematics is a live and growing subject. They have heard of the *new mathematics* and they turn to the National Council for guidance and help in connection with their own local programs. The increasing volume of correspondence received by our Washington office, the steady increase in attendance at regular meetings of the Council and the rapid growth in our membership reflect the confidence of mathematics teachers in the organization. During the past five years, for example, the membership of the Council has approximately doubled. The rate of increase has accelerated and during last year 4,297 new members were added to the membership rolls. They look to the Council for leadership and one major purpose of this meeting is to suggest services through which the Council can most effectively meet this leadership obligation. Four such services have been proposed by members of the Planning Committee. These will be presented to the conference as indicated in the agenda. A liberal amount of time is then provided for an evaluation of these proposals by members of the conference, for the suggestion of other activities appropriate to the leadership role of the Council and for ideas as to how those services on which there seems to be substantial agreement can be translated into action. We invite wide participation in the discussion and it is our hope that by discussing these problems together and exchanging our ideas about the future of mathematics education we can collectively arrive at some guidelines that will help us determine policies which will provide direction for future Council activities to the end that it will serve as a more effective instrument in meeting the needs of its members and all those interested in mathematics education."

Providing for the Gifted Child

ALICE PANEK
Rocky River, Ohio

THE GIFTED CHILD is that endowed individual, who, according to Dr. Lewis Terman of Stanford University, is generally superior in all physical traits, well-adjusted socially, and has a unique superiority over the average in intelligence—"the ability to carry on abstract thinking, to use abstract symbols in the solution of all kinds of problems." This includes about 1 to 2% of the population having I.Q.s of 130 and above.

What have educators done for these eager minds, these children whose interests are many-sided and spontaneous, who have many hobbies, who have an unusual ability to generalize, to see the parts as well as the whole, who can make a critical evaluation, and who are generally quick and alert?

Only within the present century have we learned enough about the nature of human abilities to recognize the fact that most of the potential talent of preceding centuries had not been developed and that at present, at least one-half of our best human material is not developed to anywhere near its capacity (12).

Almost every study shows that gifted children are offered little that is mentally or educationally provocative by the subject matter of the grades (24). As a matter of fact, most of them are kept at work two or three full grades below the level of achievement they have attained in standardized achievement tests (3).

To be specific, how well do gifted children score on standardized arithmetic tests? Studies show that this group on the whole is above average in arithmetic reasoning and about average in arithmetic computation. When programs are heavily weighted with routine computation, many bright children do less well that the average, for they seem to dislike prolonged drill on a subject which they feel they have mastered.

But all curriculum areas, especially the social studies and science make a heavy demand upon arithmetical knowledge and abilities. It has been shown that the ability of pupils to deal with quantitative concepts in these fields of study is very limited.

World War II heightened our recognition of the need for implementing new mathematical techniques to solve the multitudinous technological problems of the 20th century. Surely, these will challenge even the most gifted. Are they being prepared to cope with these demands upon their skills?

"Mathematics—Queen of the Sciences and Arithmetic, Queen of Mathematics" (C. F. Gauss) has been a curricular orphan in some otherwise modern schools because of the unfamiliarity with methods and techniques of teaching it functionally. Too long have courses of study been wholly dependent on a single textbook, that had as a major objective "to get through the book." Then, there are those administrators who think arithmetic would suffer by being "watered down" by integration with other subjects, and others who consider it a minor tool in the curriculum (11). Schools must organize instruction in arithmetic to deal effectively with the need for presenting the subject meaningfully to all students and especially to those for whom it is almost indispensable in the pursuit of a higher education.

It has been found that improved techniques in teaching arithmetic should include, for heterogeneous classes, recognition of variation in length of time for learning, (the bright child grasps ideas quickly), sections within a class (grouping), flexible assignments (varied and more difficult work for the gifted), larger blocks of time, part to be used for supervised study (many gifted children have poor study habits), and closer correlation between mathematics, science,

and allied subjects (bright children will often concentrate on their forte, and neglect routine studies).

There should be emphasis on experimentation, skills of investigation, independent work, a high standard of accomplishment, cooperative planning, and activity to provide leadership training, first-hand experience, a flexible procedure, concern with community responsibility, and individual help from the teacher in a true enrichment program.

To have enrichment activities in a heterogeneous classroom, it is desirable to have small classes, ample and varied instructional materials, flexible curriculums, and versatile teachers. It is also suggested that teachers be kept in charge of a class for a longer period of time to become better acquainted with their pupils.

Enrichment may be introduced through projects, experience units, creative work, participation in community activities, differentiated assignments, special interest clubs, and extensive reading. Listed below are a few suggestions on ways of supplementing the regular classroom work for the pupils who excel, listed under broad objectives of the primary and upper elementary grades in arithmetic.

In Primary Grades

NUMBER CONCEPT. Make a scrapbook of pictures of people who use numbers in their work, i.e., postman, storekeeper, telephone operator, etc.

Make a 100's chart (numbers from 1 to 100 arranged by 10's) on squared paper. This can be used in a variety of ways, to locate specific number, count, etc.

Make large dominoes of cardboard and play a regular game with them, or use them to show number.

Make checkerboard poster of 1's, 2's, 3's, showing alternate squares with shoes (2), silverware (3) etc., and the numbers used.

COMPUTATION. Make ring toss game, using cut-down broom handle and heavy linoleum with hoops of coat hangers, covered with strips of cloth or heavy rope. Keep score.

Make colored leaves (authentic copies of common tree leaves) and put combinations on them, without the answers. Put small circle of adhesive tape on back of each one and scatter on flannel board. Pupils take turns identifying as many facts as possible.

Have a quiz program using arithmetical terms.

MEASURE. Estimate length of familiar objects and check by various instruments, i.e., string, ruler, yardstick, etc.

Use a large cardboard thermometer to measure progress in arithmetic or spelling.

Lay out proposed garden plot on gym or room floor with chalk, using yardstick, rake handle, or by stepping off.

Make calendar of the whole year to indicate birthdays of each member of the class. Compute ages in years and months.

FRACTIONS. Make clay or plasticene objects (pears, apples, cake) and cut in fractional parts.

Draw pictures showing how fractions occur in child's life, i.e., half-hour, half-pint, quarter, etc.

Make attractive designs using fractional parts of squares through color, i.e., one-fourth red, one-fourth blue, etc.

GRAPHIC REPRESENTATION. Keep bar graph record of number of children present and absent for a week.

Make a map of own school area, or make a simple plan of own yard.

Illustrate the number of days it took to build a pioneer cabin and how long it takes to build a modern house. Discuss the reason for the difference.

GEOMETRIC CONCEPTS. Weave simple geometric designs on a simple loom using rayon loops of bright colors.

Make chart showing which familiar objects are circles, i.e., ring, sun. Books, bricks, desks are rectangles, etc.

Make table cover of unbleached muslin, circular or square, decorated with repetition of geometric design for gift.

PROBLEMS. Construct supermarket and trading post, showing differences between them in goods, money, displays, etc., and be able to tell the reason for it.

Emphasize number element in science experiment, i.e., planting bean seeds, counting and measuring accurately, keeping accurate record of events, etc.

In Upper Elementary Grades

NUMBER CONCEPTS. Make an Arithmetic Dictionary of terms used and their meaning with illustrations where possible.

Compare the number of states today and at other critical times in history, i.e., during Civil War, in 1776, etc.

Compare the division of degrees of various types of thermometers.

Make a mural showing ways of counting, using fingers, markers, stones, etc.

Go to observatory to see how stars are counted.

Visit post office to see sorting of mail to various areas, streets, and homes.

Make a series of flags of the United States, showing different arrangement of stars.

Make a chart of Roman and Arabic numerals, showing relative values.

Make own abacus to show place value from tinker toys, macaroni, washers, Life Savers, etc.

COMPUTATIONS. Learn how to compute use of electrical power, cost per KWH, and amount used. Cost of operation of various household appliances could make an attractive poster.

Find the cost of night and day rates of phone calls to a specified city. Compare them, also find reason for difference. Plan a picnic menu, read the food ads, average the cost per pupil, collect the money, do the shopping, and consume.

Write a short report on "WHAT I WOULD DO WITH A MILLION DOLLARS!" disposing of each amount and totaling it.

Use mail order catalogue to order adequate, poor, or lavish wardrobe for school, compare costs.

Plan a trip to foreign countries and compare cost of various means of transportation.

MEASURES. Give report or make a poster showing history of measure, foot, rod, inch, etc.

Compare face, actual, and relative value of coins in a coin collection.

Find out about various kinds of measure used in transportation, i.e., knots, miles, kilometers, etc.

Make a display of dry measure using bushel, half-bushel, peck, quart, and pint baskets. These may be painted, labeled and arranged into an attractive wall display.

Make a mental estimate of contents of can, bottle, etc., and check.

FRACTIONS. Read stock market quotations and discover what the figures mean. Study the history of football to discover reason for division of time, field, etc.

Make a study of firearms and find the meaning of calibration.

GRAPHIC REPRESENTATION. Make chart showing cost of lunch in cafeteria and packed lunch.

Show class progress in elimination of errors in spelling or arithmetic using a ribbon chart.

Use abacus to show cost of mailing a letter today and 100 years ago.

PERCENTAGE. Figure own or favorite player's batting average. (Relate to per cent.)

Compute rate of interest on bank account at various current rates.

Make a study of percentage and its origin in our number system.

Show distribution of average American budget. Compare with average income of local area.

Show how much of a dollar common percentages are, i.e., 4% is 4¢, 10% is 10¢ of a dollar.

Study methods used in gathering and tabulating data in public opinion polls.

Make a study of population of local community or state, to show percentage of native and foreign born, or voters and non-voters.

GEOMETRIC CONCEPTS. Study geometric forms in primitive art forms, i.e., hieroglyphics, etc.

Study various architectural types, Moorish, Greek, Modern, etc., and make slides showing geometric concepts predominating. Cut geometric designs in potatoes or carrots and use them with tempera or ink to make attractive wrapping paper.

PROBLEMS. Use as a theme for a study, the part arithmetic plays in man's control of nature, showing the importance of number in agriculture, forestry, fishing, etc.

Study the importance of number in economic competence, which occupations require no mathematical skills, which the most. Prepare a career day program to show what was learned.

Study information about proposed changes in the calendar and the reasons for them.

Study the importance of U. S. Savings Bonds and the part they play in our economy.

Discover the part the stock market plays in our economy. Who are the stockholders? What is done with the money?

Study origin and need for the International Date Line.

Become aware of number in the daily news. In how many different ways is number used to interpret the news?

Become aware of fun with and about arithmetic! Make a display of comics and jokes about numbers.

Consider importance of dams. Who should finance them and why?

These suggestions are intended to supplement a text or course of study to insure the gifted child's acquisition of basic skills and understandings. Many courses of study now contain excellent lists of activities for the development of number meanings. However, care must be observed in their use. They must not become mere devices for manipulation that do not contribute to insight and functional use of numbers. Number ideas and concepts should emerge from the situations in which they are being used to increase the exactness in thinking and carrying out of useful activities.

It has been found that it is a mistake to allow bright pupils to do minimal amounts of fundamental work. Unless there is repetition, learning is not permanent. Talent seldom flourishes without practice, and arithmetic is no exception.

Drill on skills should be short and intense and done on an individual basis or in small groups. There should be a limited amount of work in a workbook and emphasis on mental arithmetic.

Dr. Harry Baker, director of the Psychological Clinic of the Detroit Public Schools, pointed out that the products of high education are judged mercilessily by an unsympathetic business world, chiefly in terms of ability to figure correctly and to write legibly, whether these be good criteria of intelligence or not. With little extra effort, these "evidences of practical education" (2)

can be perfected to a surprising degree. Here then is another area which must be considered by the skillful teacher.

President L. A. DuBridge of California Institute of Technology stated, "Properly presented and properly taught, mathematics is an exciting adventure. What has made it seem hard is the endless procession of dull and useless problems which are normally taught. It is an illusion to think that mathematics can be taught by teachers who do not know any math, or are only a chapter ahead of the student. It is a subject that comes alive only with years of study and can be conveyed in simple and exciting ways to students only by those who have themselves caught its true spirit".

Yes, "whether a school is large or small, whether its program is the most complex and expensive, or the simplest and least costly—the final success of its efforts to aid its bright pupils will depend upon one thing—the kind of men and women who make up its faculty" (24).

Indeed the best program will ultimately meet with failure, unless each teacher recognizes the privilege and the responsibility of preparing for their rightful places in society our most precious resource, our gifted children.

Bibliography

1. Adams, Fay, *Educating America's Children.* N. Y.: The Ronald Press, 1946.
2. Baker, Harry, *Characteristic Differences in Bright and Dull Pupils.* Ill.: Public School Publishing Co., 1927.
3. Baker, Harry, *Introduction to Exceptional Children.* N. Y.: Macmillan Co., 1953.
4. Burington, R. S., "Contemporary Applications of Mathematics," THE MATHEMATICS TEACHER, Vol. XLIX, #5, May, 1956.
6. Education Policies Commission, "Education of the Gifted." Wash., D. C.: N.E.A. & The American Assoc. of School Administrators, 1950.
7. Euclid, Ohio, Board of Education, "Teaching Guide to Grade 4." Euclid Curriculum Bulletin 5, 1954.
8. Gesell, Arnold, *Exceptional Children and Public School Policy.* New Haven: Yale University Press, 1921.
9. Hall, Jack, *Solving Arithmetic Problems Mentally.* Iowa State Teachers College, Cedar Falls, Iowa, 1955.
10. Hall, Theodore, *Gifted Children, the Cleveland Story.* Cleveland: World Publishing Co., 1956.
11. Harding, Lowry W., *Functional Arithmetic Photographic Interpretations.* Dubuque, Iowa: Wm. Brown Co., 1952.
12. Havinghurst, R. J., Stevers, Eugene, and De Haan, R. F., "A Survey of the Education of Gifted Children." The Committee on Human Development. Chicago: University of Chicago Press, #83, Nov., 1955.
13. Hildreth, Gertrude, *Educating Gifted Children at Hunter College Elementary School.* N. Y.: Harper & Bros., 1952.
14. Kuhlen, Raymond, and Thompson, George, *Psychological Studies of Human Development.* N. Y.: Appleton-Century, Crofts, Inc., 1952.
15. Lee, J. Murray and Lee, Dorris May, *The Child and His Curriculum.* N. Y.: Appleton-Century, Crofts, Inc., 1940.
16. Moore, Lillian, "Challenge of the Bright Pupil." THE MATHEMATICS TEACHER, 34: 155–157, April, 1941.
17. Newland, T. Ernest, *The Gifted.* Review of Educational Research, Vol. XXIII, #5, Dec., 1953.
18. Ohio Schools, Official Journal Ohio Education Assoc., *Cooper-action Proves Its Worth.* Vol. XXXIV, #5, May, 1955.
19. Osburn, W. J. U. and Rohan, B. J., *Enriching the Curriculum for Gifted Children.* N. Y.: The Macmillan Co., 1931.
20. Reeve, W. D., "The Problems of Varying Abilities Among Students in Mathematics." THE MATHEMATICS TEACHER, Vol. XLIX, Feb., 1956.
21. Terman, L. M. and Oden, Melita H., *The Gifted Child Grows Up.* Stanford, Calif.: Stanford University Press, 1947.
22. Thompson, George, *Child Psychology.* Boston: Houghton Mifflin Co., 1952.
23. Witty, Paul, *Helping the Gifted Child.* Science Research, Inc., Chicago, 1951.
24. Witty, Paul, *The Gifted Child.* Ed. American Assoc. for Gifted Children. Boston: Heath & Co., 1951.
25. *Yearbook of National Society for the Study of Education.* Fiftieth, Part II: The Teaching of Arithmetic. Chicago: University of Chicago Press, 1951.

EDITOR'S NOTE. It is most heartening to note the renewed interest in our "gifted children" because it is upon them that we must depend for the higher levels of thinking in our advancing civilization and for our own national security. Perhaps it is equally important to consider *how* we guide them as well as *what* we do with them in our elementary and secondary schools. Mrs. Panek has listed and described at both the primary grade and upper grade levels some of the avenues of study that might be used with these pupils. The resourceful teacher will supplement this list. The important point is that we must do something other than have them rush through the "minimum essentials" of arithmetic. They must learn the fundamentals and learn to perform but they should also learn to think, to explore, and to pioneer with ideas in arithmetic. For them it is perhaps more important to have practice in the thinking-discovery aspects of learning than in the practice-drill phases of learning to compute.

"Plus" Work for "Plus" Pupils

EUNICE LEWIS and ERNEST C. PLATH*
The University of Oklahoma, Norman

ONE PLUS ONE EQUALS "10" for the members of a special arithmetic class at the University School, College of Education, Norman, Oklahoma. Of course, the members of this class were working with a number system of base two, commonly referred to as the binary system. Students also readily stated that three plus three equals "12" if the base is four. Changing the base number was not only fascinating to these highly talented fifth and sixth grade youngsters, but also provided a launching platform for the development of complicated formulas (patterns to them) which are normally developed in a second year high-school algebra course.

Well aware that there are several ways of meeting the needs of talented youngsters such as grade skipping, enrichment within the traditional classroom structure, and by separation and specialized instruction, it was decided that separation and special instruction was probably the most promising method.

Basic Hypotheses

It was surmised that:

1. Children with high scholastic ability are capable of gaining an insight into numerical relationships at a much higher level than is normally presented on the grade level which matches their chronological age.
2. Under the guidance of an experienced teacher with a strong background in mathematics and professional education, high-ability youngsters are capable of developing generalizations concerning numerical relationships even though exact verbalization on the part of the student is not possible until later.

Basis of the Selection of the Children

Toward the latter part of the first semester of the 1957–58 school year, eleven children (three fifth graders and eight sixth graders) were selected to be taken from their regular classroom for special mathematics instruction for approximately thirty to forty-five minutes each day. This was to be in addition to their regular mathematics class. The children selected had functional intelligence quotients of 130 upward as measured by the California Test of Mental Maturity (Form SF) and at least a 6.5 grade achievement level on the California Achievement Test (Form AA-Elementary), which had been administered in September of 1957. Scores on these tests are reported in Table 1 as follows:

TABLE 1
TEST SCORES OF SELECTED STUDENTS

Pupil	Arithmetic Reasoning	Arithmetic Fundamentals	Total Arithmetic	Calif. Test of M.M. (IQ)
	Grade Placement Scores			IQ
A5	7.2	6.3	6.5	130
B5	8.1	7.4	7.6	142
C5	7.6	6.4	6.7	131
A6	8.3	7.1	7.4	146
B6	8.3	7.2	7.5	140
C6	8.3	7.1	7.4	142
D6	6.7	6.7	6.7	130
E6	8.7	6.5	7.0	133
F6	7.4	7.0	7.1	143
G6	10.0	8.7	9.0	153
H6	7.6	7.3	7.4	135

* Dr. Plath has moved to The University of North Dakota since this study was completed.

Parents Interviewed and Informed

Parents of the children were interviewed and each parent was shown the group's test results with the names of pupils coded, which informed them of their youngster's standing within the group without identifying the other pupils. The parents were asked to give their permission for their child to participate in the experiment and to report any reactions that their child had at home concerning the instruction the students were receiving. Parents were also asked to watch certain social developments that might take place as a result of the experiment, especially any development of superior feelings on the part of their child. Homeroom teachers were also alerted to see if they could ascertain whether there was any difference in the reaction of other youngsters in the class to the experimental students. All parents gave their permission for their children to participate in the experiment with it understood that it was highly possible that this experiment might terminate at the end of the semester. The parents were assured that their youngsters would receive the same instruction in the regular classroom as the youngsters not selected for this special experiment.

Students Informed of the Experiment

After the parents gave their permission, the authors met with the selected youngsters and informed them of the nature of the experimental program. The students had the following comments, suggestions, and questions:

"I want to know what we are getting into before I make up my mind."

"Are we going to use a textbook, or a workbook?"

"Will we get into algebra or geometry?"

"Will we have to work a lot of problems?"

"Are we going to learn the principles rather than just work problems?"

"How will this affect our lunch schedule?"

It was explained that: (1) this was to be a different program than the one which the students had been experiencing; (2) that the assignments would not involve drill work to any great extent; (3) that there would be challenging problems to tax the thought processes; (4) that there would be no set curriculum which would have to be accomplished. The students seemed satisfied and were given this problem to be considered before their first class session: "What is the sum of the first 60 odd numbers?" Immediately, the students reacted by giving possible methods of attack and seemed eager to begin their first session on Monday. After the children left the room, the following observation was made with regard to the first meeting of the group. The developmental level displayed by this group was astounding—many adult-like observations were made and the enthusiasm for the experiment was high.

The First Session

At the first regular session of the special arithmetic class, the teacher asked for possible solutions to the problem which had been posed on the preceding Friday, "What is the sum of the first 60 odd numbers?" Immediately, five of the students volunteered solutions. One observed that the sum would be divisible by sixty, while two felt that the sum was connected with the sum of the numbers; one, three, five, seven, and nine; they detected a repeating pattern. Another student who obviously *had* the solution, pointed out that the square of 60 was the answer. Still another disagreed, feeling that 60 *times itself* was correct. This led to the clarification of the term "to square." The teacher next showed the symbol which means "to square," and this led to a general discussion of exponents and their function. The group was then asked to find the number 2 (two) to the twelfth power for their next session. In order that all of the group might discover the formula for finding the sum of the first 60 odd numbers (or the sum of any given number of odd numbers), the

teacher wrote a table in the following manner:

Order	*Sum*
1st number	1
2nd number	4
3rd number	9
4th number	16

With this hint, most of the group quickly realized that the relationship between the order and the sum was that the sum equals the order squared. Four of five students saw the relationship only vaguely. One student observed that by extending this we could construct a table by which we could "read off" squares and square roots. Another remarked that he imagined that must be the way such a table was constructed which he had seen in his big brother's mathematics book.

Some of the group wondered about even numbers and their relationships, so the teacher used this opportunity to assign the following: "What is the sum of the first 20 *even* numbers?"

Even though the first session lasted well into the students' lunch hour, none of the group wished to leave. Some remained to talk about the graph that was on the chalkboard in the room for a high school mathematics class. Enthusiasm was high and remained high for the semester with the youngsters developing an understanding of numerical relationships until they reached a pinnacle of understanding that was purported to be impossible at the beginning of the experiment.

Extended Work with Numbers

During the course of instruction, the students covered many phases of mathematics, the most popular being in the area of number relations. Even though some time was spent in working with geometric relations, the casual interest in these as compared to the excited interest in the number relations was quite marked. Consequently, most of the work was concentrated on looking for "patterns" which developed when the students worked with numbers in various ways.

It was interesting to note the fascination that the group had for symbols. After working with the sum of odd and of even numbers, the question was asked, "What is the difference between an odd and an even number?" The discussion developed from the idea that even numbers were *like* 2, 4, 6, etc. and odd numbers were *like* 1, 3, 5, etc. to "even numbers could be divided by 2 evenly, and odd numbers could not be." Later, the students discovered that any number could be made even by multiplying it by 2. This led to the use of a symbol to hold the place of this "any number." The symbol which they chose was n, with $2 \times n$ representing the even number. One pupil observed that by adding 1 to an even number an odd one is formed, so, he went on to add that $2 \times n + 1$ would be a pattern for an odd number. Another pupil quickly remarked that 1 could be subtracted from the even number to make an odd number.

Interest was manifest in constructing sets of numbers by using a "jumping number." These sets are recognized by second year algebra students who use a more sophisticated language to describe them as arithmetic progressions. Many exercises were given with sufficient break-down of the processes involved in constructing these progressions to give opportunity for the group to recognize a pattern. For example: the students were asked to continue to construct a set of numbers which started like this, 2, 5, 8, 11. The numbers 14, 17, etc., were immediately supplied. A student observed that we "Get from 11 to 14 by the 'jumping number,' 3." The teacher then posed the question, "What is the 18th number?" After a brief moment, one student replied that it was 53 while the others diligently began to set up a table as shown below:

1st number is 2	 2
2nd number is . . . . . 2+3	. . . . . 2+1·(3)
3rd number is . . . . . 2+3+3	. . . . . 2+2·(3)

Then it was observed that the numbers could be written as:

1st number is 2

2nd number is.....$2+(2-1)3$
3rd number is.....$2+(3-1)3$
etc.

One student said that he could write the eighteenth number in this way, $2+(18-1)3$. It took a very short time for the students to come up with the following statement accompanied with the pattern, "any number can be found by multiplying the 'jumping number' by one less than the number, then adding the beginning number." The pattern in symbols which they developed was: nth number is $fn+(n-1)\cdot jn$. This is only one example of some of the areas of investigation which the students undertook and the method used in assisting them in making interesting discoveries which proved very exciting to them.

One interesting observation which was made was the expression of creativity among the group shown in their interest in new ideas and in their use of unconventional symbols, as can be noted in the above formula, recognized as the algebra formula, nth term $=a+(n-1)d$, for finding a particular term in an arithmetic progression. Also, when the number system was extended from the Natural Numbers or counting numbers to the Integer Set, they used Σ and π to denote the opposite directions and $\odot$ as the symbol which divided these two sets. One might expect that from normal experience with games they might have chosen the conventional ones, $+$, $-$, and 0.

Even though the children and their teacher gained a great deal of fame locally and very little fortune with regard to their special mathematics class, there was little to indicate that the rest of the youngsters considered them "egg-heads" or that the youngsters themselves developed a feeling of superiority. This was true even though the special class was used for two demonstrations before large groups visiting the campus of the University of Oklahoma plus a television appearance plus front-page publicity in the local paper. As the finale, the special arithmetic group held a special demonstration for their parents in which they reviewed some of the areas which had been covered during the semester and took up a new problem-area or two to show how they responded to new situations.

At the luncheon which followed, parents were given a progress report by the authors and some observations by the graduate student who kept the daily log of the developments of the program. Parents were unanimous in their enthusiastic support of the experiment and were very anxious to have their children continue such a program. Some expressed the desire that highly talented students be given instruction in other fields as well.

Testing the Success of the Program

During the last sessions of the class, the California Achievement Test—Arithmetic, Elementary; the Iowa Test of Basic Skills (Form I); and the Davis Test of Functional Competence in Mathematics (Form AM) were administered for comparison purposes. Results of these tests are shown in Table 2 and 3.

It should be noted from a comparison of data contained in Table 1 and Table 3 that all of the youngsters made a definite gain during their semester of instruction in the special arithmetic class. It was found that neither the California nor the Iowa Test of Basic Skills adequately measured most of the youngsters in the group. As a result of these tests and observations made of the ability of the youngsters to solve problems of considerable abstraction, the authors have concluded tentatively that: (1) children with high scholastic ability are capable of gaining insight into numerical relationships at a much higher level than is normally presented to them on a grade level which matches their chronological age; (2) under the guidance of a well-prepared, experienced teacher, high ability-level children are capable of developing generalizations concerning the numerical relationships even though the exact verbalization on the part of the children is not possible until later. It was found that

TABLE 2

ACHIEVEMENT SCORES OF SPECIAL ARITHMETIC CLASS AFTER A SEMESTER OF INSTRUCTION, MAY, 1958

	California*						Iowa Test of Basic Skills**			
	Sept. (Form AA)			April (Form W)			April (Form 1)			Davis Test***
	Reasoning	Fund.	Comp.	Reasoning	Fund.	Comp.	Reasoning	Fund.	Comp.	
A5	95	90	90	99+	99+	99+	99+	98	97	59
B5	99+	99+	99+	99+	99+	99+	99	99	99	59
C5	95	90	90	99+	99+	99+	99+	99+	99+	38
A6	95	75	99+	99+	99+	99+	99+	99+	99+	87
B6	95	80	85	99+	99+	99+	99+	99+	99+	74
C6	95	75	85	99+	99+	99+	99+	99+	99+	98
D6	75	70	70	99+	99+	99+	99+	99+	99+	59
E6	95	70	75	99+	99+	99+	99	97	98	44
F6	85	75	75	99+	99+	99+	99+	99+	99+	87
G6	99	90	95	99+	99+	99+	99+	99+	99+	91
H6	90	80	85	99+	99+	99+	99+	99+	99+	44

* California Achievement Test—Arithmetic, Elementary.
** Iowa Test of Basic Skills, 1957, Form 1.
*** Davis Test of Functional Competence in Mathematics (Form AM). These percentiles are based on National norms for the ninth grade.

this group of fifth and sixth grade youngsters was capable of developing formulas usually presented to juniors in high school.

The latter conclusions raised an interesting question with regard to quality of instruction and mathematical background of classroom teachers. If youngsters of this calibre are not separated for special instruction, does the average elementary classroom teacher have the depth of mathematical knowledge necessary to provide the motivation and the instruction necessary to challenge youngsters of high mathematical ability? In order to answer this question, the authors turned to some data and made comparisons between the mathematical competence of the experimental group youngsters and college juniors and seniors who were enrolled in a course in Methods of Teaching Arithmetic. The data used for comparison are presented in Table 3 with regard to results of the Davis Test of Mathematical Competence.

In comparing the special arithmetic group of fifth and sixth graders to the college group of future elementary school teachers according to the twelfth grade norms of the Davis Test of Functional Competence in Mathematics, it was found that the 52 college students had a mean percentile of 52 and a class median percentile of 53. The range of ability of these 52 students (college) was from the fourth percentile to the ninety-seventh percentile. The eleven fifth- and

TABLE 3

A COMPARISON OF THE SPECIAL ARITHMETIC AND COLLEGE GROUP

Davis Test of Functional Competence in Mathematics (12tk Grade Norms)							
College Group				*Special Arithmetic Group*			
Number	*Range*	*Mean*	*Median*	*Number*	*Range*	*Mean*	*Median*
	(Percentile Scores)				(Percentile Scores)		
52	4–97	52	53	11	15–77	39	39
				11	38–98*	69*	59*

* Ninth-grade norms.

sixth-grade youngsters ranged from the fifteenth percentile as compared to twelfth grade norms. The mean and median of the experimental group was the thirty-ninth percentile. If, however, the special arithmetic group were rated on ninth grade norms, they would range from the thirty-eighth through the ninety-eighth percentile with a mean percentile ranking of 69 and a median percentile of 59. According to this comparison, eight of the eleven or 73 per cent of the special arithmetic class would be above the college percentile mean. From this data, it might be concluded that special attention should be given to mathematical competence of prospective elementary teachers if the needs of talented students are to be met or that these highly talented youngsters should be given special instruction under the guidance of an experienced teacher with a strong background in mathematics and professional education.

Summary and Conclusions

What does one plus one equal? That depends on the number base. The fifth and sixth grade students selected for this experiment had little difficulty understanding this mathematical relationship or in developing formulas usually presented at a much higher grade level. The selected students and their teacher amazed mathematicians and educators with their performance in educational gatherings and television appearances. Parents of these selected students were highly enthusiastic concerning the special instruction their children received in mathematics and expressed a desire to extend this special instruction to other areas.

Even though the experimental group was small and definite conclusions cannot be made, there is a very strong indication that:

1. Separation and specialized instruction for talented youngsters is a promising procedure for providing for giftedness.
2. It is possible to select highly talented youngsters with a minimum of error by the use of group standardized tests.
3. A critical examination should be made of the self-contained classroom to determine if it is possible for any one teacher to attain the degree of competence in all subject-matter areas necessary to provide instruction to talented students on the level they are capable of performing.
4. Students are capable of gaining insight into numerical relationship at a higher level than is normally present on the grade level which matches their chronological age, provided, of course, that they are under the guidance of a competent teacher.
5. Parents are willing to have their children participate if they are informed of purposes and procedures.

EDITOR'S NOTE. Here again we have evidence that our more able pupils when stimulated and led by competent teachers are able to develop insights and discover "patterns" and relationships far beyond the level ordinarily assigned to their age group. The authors indicate that it is the thinking that is important and that we need not express this thinking and the final generalizations in conventional symbolism. We should strive for a reasonable sophistication of verbalism but this need not be the same as that which we use several years later when students have reached a greater maturity.

There are many avenues of exploration that a teacher might open up to his gifted pupils. The authors suggest that a higher level of training might well be indicated for a teacher who undertakes such work. This need not be training formally acquired in an advanced class in mathematics. It can be acquired by independent study. Why not form "Teacher groups" that study mathematics? Certainly someone both willing and able can be found who will help a group in the harder spots.

Topology

(*Concluded from p. 279*)

Before taking that adventure however, a final accounting was taken of what we had learned in mathematics and science as the IGY period came to its end. Of all the discoveries and experiments undertaken by scientists in all the world, our own encounter with the Mobius Surface was the most dramatic because it made us think about the possibilities of unusual properties of space still waiting to be discovered by *thinking* minds.

A Gifted Underachiever in Arithmetic

A Case Study

ANN F. ISAACS*
Cincinnati, Ohio

THE DATA PRESENTED in the history which follows is of interest from three points of view. First, a majority of the reports in the literature on the "gifted" devoted to the development of children with superior endowment stress the fact that superiority is not unitary. Gifted children tend to exceed the norm in all areas. A study of gifted twin boys (2) illustrates this point of view. Secondly, various studies have attempted to show that students of education are not comparable in endowment to students in other fields. Wolfle (3) in exploring some thirty fields of endeavor has shown that the combined fields of physical education and general education contained the fewest number of cases on the high end of the distribution curve. The subject of this study tends to be an exception to both of the above since she has a problem, and though of very superior intelligence is the offspring of educators, also of very superior intelligence.

Thirdly, information gleaned in connection with gathering material for this study may point to an area which merits further investigation. Interviews with art teachers and a dancer of world fame, Jose Limon (1), whose initial training was in art, combine to reveal that difficulty was experienced in the study of arithmetic. Information gathered for this case tends to follow a similar pattern.

Identifying Data

Nancy Jo is an attractive blue-eyed, blond-haired and well-built child. At the time of this study she was ten years and ten months old, attending the sixth grade of a suburban school located in the East where she was born. It was at the suggestion of the school principal that remedial work in arithmetic was begun with the child. At the close of the school year Nancy Jo will be entering the junior high school. The elementary school the child now attends does not go beyond the sixth grade. Both the school and the child's parents are eager that she begin this next segment of her education with as firm a foundation as possible. The report card brought home was acceptable in all respects being composed of A's and B's chiefly. A's were obtained in Art, Music, Reading, and Spelling, B's in Science, Social Studies, and Physical Education. The exception was in arithmetic where a D had been received.

Physical History

Nancy Jo was a full term baby, with normal birth. She was bottle fed, and grew along the expected developmental norms, trippling her six pound birth weight at the end of the first year. She walked and talked at one year, and by the time she was two was talking in nine word sentences. She attended nursery school from the time she was two, the school principal accepting her because in her estimation, she was equal to the average three or three and one half year-old child. She had measles and chicken pox at three. In addition up until the past year she was troubled with one cold and sore throat after another. This resulted in frequent absences from school, at times for periods as long as ten days to two weeks. Mother is of the opinion that it is likely Nancy Jo's arithmetic difficulties began some time during this period. Interestingly the child herself traces her dislike of the subject back to the

* Ann F. Isaacs is Executive Director of The National Association for Gifted Children and Editor of *The Gifted Child Quarterly*.

second and third grades when mother again resumed teaching on a part-time basis.

Overtly Nancy Jo is not a child one would single out in the classroom as needing special attention. The twinkle in her eye, and the quick smile, unite to create the impression that she is a child whose problems are few. In general she gets along well with her teachers and has from time to time been the favorite pupil. The fifth grade language arts teacher who had been teaching school some twenty years was of the opinion that Nancy's language facility outshone all pupils she had ever taught.

She and several girl friends take turns visiting and over nighting at one another's homes. When Nancy Jo has a guest, she then sometimes excludes younger sister, much to the distress of the six-year old. More often she is thoughtful and considerate of sister, reading to her and playing games with her. Occasionally surprises are given to Mommy and Daddy, by way of little presents they make together, or preparing and serving breakfast to them.

With the family she enjoys gardening, going on trips, swimming, plays, concerts, lectures, museum visits and calling on relatives.

School History

In the eyes of the parents this child has always been an underachiever, making grades that would be chiefly characterized as mediocre. She started school about one-half year under age, since her birthday was near the end of the year, but has been fortunate in her social adjustment, in that there have been many classmates who were in a similar situation and who came up through the grades with her. The parents considered having her tutored, but have been reluctant to deprive her of her little free time since (a) frequent colds made it seem she needed the rest rather than remediation, (b) there was a slow but steady improvement in her grades sans remediation, (c) a feeling that the school ascribed to a philosophy of giving poor grades to motivate the children to worker harder, which for this child was obviously not effective.

She has difficulty in completing her work, if it is something she has little interest in doing. This is in direct contrast to the picture given of the child in preschool, which described her as of extremely long attention span, holding with one activity for as long as several hours, even though others in the group had long lost interest and gone to other activities. Though she has seen teachers scolding boys and girls for finishing their work too soon, so that now few do this anymore, she still wants to be a teacher, because teachers can help lots of boys and girls. She indicated she would only praise those who did things right, and demonstrate to those who did not understand, how they could improve. She further would try not to have "pets." Having been the pet in some classes, she knows how it feels to be both on the inside and the outside of the "pet circle."

Psychological Test Results

On the revised Stanford Binet, Form L, administered to Nancy Jo when she was ten years eleven months, an I.Q. of 174 which gave her a mental age of 19 years and 3 months was obtained. Her basal age was established at year fourteen. At the Average Adult level, the ingenuity problem was missed, at S.A. I, the mincus completion, at S.A. II, the reconstruction of opposites was missed, while at S.A. III credit was earned for only the paper cutting. Form M, administered two years previously, yielded an I.Q. of 178 for Nancy Jo.

At the same time other tests administered were:

Test	*Grade*
GATES READING SURVEY, TEST FOR GRADES 3 to 10, Form II, yielded an average reading grade of	9.4
NEW STANDARD ACHIEVEMENT TEST	
Arithmetic Reasoning	4.9
Arithmetic Computation	4.3
METROPOLITAN ACHIEVEMENT TEST, Intermediate Battery, Form A, Arithmetic Fundamentals	4.7
UNIT SCALES OF ATTAINMENT, Primary Division, Grade three, Form A, Fundamental Operations	4.9

Her difficulties were further analyzed by means of the Buswell-John Diagnostic Chart for Fundamental Processes in Arithmetic. She was able to do most of the addition examples, but had great difficulty with multiplication, and was unsure of many of her arithmetic combinations. Division and subtraction problems showed errors of carelessness.

Family History

In addition to father who is a school superintendent, and mother who is a school principal (in another school system), in the home, there is also a younger sister, Jane Ann who is six and in the first grade, and the maternal grandmother whose wonderful cooking is appreciated by all.

Time permitting, father enjoys his record collection of classical music, working in his rose garden, football and basketball, which the girls also like to watch, and reading in a wide variety of fields.

Mother in addition to her teaching enjoys sewing for the girls, painting, piano, cooking, writing (she has sold several short stories of which the whole family is proud), gardening, and just being with the children as much as possible.

Both sides of the family are of English-French extraction, with great grandparents coming to America in their youth. Father indicates he can well understand Nancy's current ambition of wanting to be a teacher, which little sister also shares. Both mother's and father's grandfathers were ministers, with a history that includes preachers and teachers a good way back. Thus father understands how Nancy is good at language arts. As for her fine art, the family tree has numerous artists on its limbs. Father has a distant cousin who wrote a book and did his own illustration. Mother has several uncles and cousins who earn their livelihood as professional artists. Father further indicated that both the child's mother and he were able students (each holds the Master's degree, and the father has the Ph.D. as well), but neither of them comes anywhere near this child in endowment. They have often had to stretch to find answers to some of the questions she seeks to know more about.

The family lives in an old frame twelve-room house, close to the school the children attend, which is in a system other than those with which the parents are professionally connected. They moved to this residence right before Nancy Jo was born. Since then they have grown into the house filling it with the children, books, music, paintings, antiques, and interesting visitors.

Social History

Nancy Jo attends Sunday school regularly and has won recognition for her art and literary efforts here. On Mondays she attends Girl Scout meetings, and has been active since the Brownie stage, four years ago. The past two summers she has attended Girl Scout camp. On Saturdays and during vacation periods, Nancy Jo attends art classes at the museum. Her teacher in ceramics feels she has something "special," executing her problems a little differently from the other students. She has several times won city wide recognition for her entries in art and literary competitions. When she was younger she took several years of piano and ballet, but did not like to practice, happily giving up the lessons instead. She loves music, but only "to listen to." She reads voraciously and widely. The past few months she has read teen-age romance stories, adventure (*The Bafut Beagles*), fairy stories, and a tenth grade world history book. The latter she got at the public library, and could not help but wonder why it was so easy if it were intended for tenth grade students.

Interviews

At the first meeting with Nancy Jo she was encouraged to write a story about how she felt about arithmetic. She indicated she was little attracted to the subject, calling her essay "I Hate Arithmetic, Always Have, and Always Will." (Both parents have had advanced courses in statistics.) She further said she could not see how such a nice person as

her teacher could teach so horrible a subject. She sees no point to arithmetic, and if it is so important, she wished she were talented in this subject instead of art.

Nancy Jo said she has wanted to do many things when she is grown, but has never thought of a career that would have much connection with the manipulation of numbers. She said some of her ambitions were quite funny. When only three for example, she wanted to have a rain-wear store, and sell umbrellas, raincoats, and boots exclusively. At five she and a friend wanted to be comedians, or comics. At six she wanted to be a nurse. At seven and eight she wanted to be a dentist so she could invent a hypodermic to give to people to prevent cavities. At nine she wanted to be a famous writer. At ten she wants to be a teacher, perhaps of art or literature. Because teachers can help "lots of boys and girls do well in school and in life."

Then the arithmetic teacher was visited. In her estimation Nancy Jo's problem was a combination of speed and carelessness. Analysis of old test papers showed errors such as omission of decimal points, and quizzes only half finished. What was completed was otherwise correct. How else might she be helped? Teacher suggested that the arithmetic text be taken home, and the problems prepared by the authors be worked through. This the parents agreed to do. The plan of treatment was then explained to the parents, and the means by which they might help was recommended. Again they readily agreed.

Diagnosis and Plan of Treatment

This is a very superior child who is functioning ably in two of the areas of her known talents, namely the language arts and the field of art. In arithmetic it appears however she is doing work that is inferior to her potentialities. In terms of her arithmetic achievement 4.6 grade, and chronological age of 9 years and 9 months, she is only one year retarded compared with her own chronological age. In terms of her grade placement, she is almost two years retarded, having been in the sixth grade for three months. However it is in terms of her intellectual endowment and arithmetic achievement that the greatest distance seems to exist. Her M.A. of 19[4] makes it seem that she is performing much below her capacity. Perhaps she may be less able in numerical computation, or less interested than in other areas, or it may simply be that mastery of the arithmetic facts is uncertain. In any case it is felt remediation will bring about some improvement in arithmetic, since specific areas requiring mastery have been located.

Treatment

The first session set the pattern for those which followed. Diagnostic tests were administered to determine the areas of difficulty and pinpointing the deficiencies which existed and which would be eliminated if possible. The period began with an art project which involved decorating the manila folder which was to contain the remedial work in the weeks to come. Secondly the practice of the arithmetic facts that were identified as requiring mastery was accomplished through working problems specifically designed, writing each one repeatedly while saying it aloud, using flash cards, and making up a story for each group of facts. Thirdly at the end of the period, those facts still unmastered were listed and assigned as outside work to be learned. Fourthly, the suggestion was given the parents that Nancy Jo be presented with grocery, hardware, and drug purchase statements and payments that they might be checked for accuracy, thus providing her with extra practice that she may be helped to see the practical use of numbers. Lastly, booklets from the Row Peterson Series on ENRICHMENT MATERIALS IN ARITHMETIC, GRADES THREE TO SIX,* by Larsen and Peterman were used

* Though from the title, the authors intended the booklets to be used as supplementary materials for good achievers, in this instance they were successfully used with this gifted underachiever.

both during the remedial work period, and assigned for outside study. These materials through stories, games, puzzles and other devices served to motivate the child, increasing her interest in numbers, while supplying an additional source of practice.

Plan

Each session included some drill aimed at achieving mastery over the precise addition, subtraction, multiplication and division facts which the child still lacked. This was done in a setting of fun and encouragement, praising the child for all achievement, however slight. Art work was to be included in each meeting and some opportunity was given to utilize the facility in language arts through the writing of a poem, story, or autobiographical essay. It was hoped some positive transference would thus be effected from the subjects, where mastery exists, to the deficiencies in arithmetic.

Evaluation was constant in order to find out the basis of motivating the subject to greater effort. The setting provided for practice with praise and satisfaction. Games and stories were used as freely and frequently as possible.

Follow Up

In contrast with the grades earned by Nancy Jo on arithmetic tests in her previous school work, which were seldom more than C's, D's, and a few F's because of work incompleted, a new picture presented itself shortly after remediation was initiated. The second and third weeks after work was begun with the child, papers bearing grades of A and 100 were brought home on her number work. Nancy Jo's explanation of this was that fractions were now being studied, and this was different from other arithmetic. Fractions were fun and so easy to do, she was one of the first to complete her work in class.

However since much of the literature on gifted stresses the fact that gifted boys and girls require little drill, one wonders if major growth strides had not already been made, since mastery of addition, subtraction, multiplication and division facts are basic in the operations of fractions.

Further insight into this child's progress is evident from the achievement test scores on arithmetic administered six months later, at the end of the school year. On the Stanford Achievement Test, Form B, the Arithmetic Computation showed an increase from 4.3 to 5.9. The Arithmetic Reasoning rose from 4.9 to 8.7, the increment was 1.6 years on Arithmetic Computation and almost four years on Arithmetic Reasoning.

When told of her progress Nancy Jo's comment was "I still think my best career in Mathematics will be to find a husband who is really good at arithmetic."

References

1. Personal interview with Jose Limon.
2. "Report of the Art Panel" in "Secondary School Curriculum Problems." *American Council of Learned Societies*, 9 (November, 1958), 12–16.
3. Witty and Coomer, "A Case Study of Gifted Twin Boys." *Exceptional Children*, 22 (December, 1955), 104–8+.
4. Wolfle, D, *America's Resources of Specialized Talent*. New York: Harper Bros., 1954, xvii–323.

EDITOR'S NOTE. Readers will note the background of information that was gathered in the case of Nancy Jo. It is sometimes very difficult to find the origin of one's dislike with a subject and frequently a dislike is coupled with lack of success in this subject. Usually one factor affects the other and it is not easy to tell which came first or has the greater significance. With most children who have difficulty with arithmetic and particularly with those of superior general ability, it is worthwhile knowing more than just their specific troubles in arithmetic. Frequently a particular motivation is needed. Only in extremely rare cases is this a disability that is so ingrained that clinical attention is required.

But let us not assume that all very bright children are potential mathematicians or that they should be directed toward this end. Bright children usually respond to the thinking, to the non-memorization, types of learning. We need some good experimentation with modes of learning suited to this type of child. We hope that Nancy Jo will find a husband who can do the ordered thinking and the necessary computations to insure an intelligently operated household or lacking this goal that she will, when necessary, learn to do these things herself.

Blocks to Arithmetical Understanding

CALHOUN C. COLLIER
Michigan State University, East Lansing

IT SEEMS REASONABLE TO SUGGEST, at the outset of this address,* that several points be kept in mind.

1. Mathematics, or arithmetic if some of you prefer, was one of the earlier subjects to be included in the elementary school curriculum and in all probability it will continue to be studied in the elementary school in the foreseeable future.
2. Mathematics deals with quantity.
3. People are continually facing problems concerning quantity which need to be solved.
4. The most effective and satisfactory solutions to man's quantitative problems are those resulting from his best thinking and reasoning. This thinking and reasoning is naturally based upon his understandings of arithmetic.

Since man lives in a continually changing scientific world in which he is called upon more and more to make judgments and take actions relative to some quantitative aspect of daily living, it becomes almost imperative that we develop arithmetical understanding. If children are really to learn, enjoy and find success in arithmetic, and in other areas of mathematics, they must know the meaning of numbers and must understand why they do in working with numbers. They need to know the "how" and the "why" as well as the "what" of arithmetic.

For 25–30 years writers in the field have been saying that computational ability is not enough; that understanding and reasoning are just as important, if not more important. In fact, some authorities in the field are quick to point out that ". . . it is the presence of the factors of meaning and understanding that raises the performance of the pupil above that of a computing machine." (1).

If we accept the statements in elementary school arithmetic textbooks, teachers manuals, and courses of study we invariably find that functional mathematical understandings and effective quantitative thinking are currently considered to be important instructional objectives.

I also believe that teachers in increasing numbers are exerting a greater and more consistent effort to make arithmetic meaningful and functional to the pupil.

With rather general acceptance among educators of a theory of arithmetic instruction which emphasizes meanings and understandings and an apparent increasing effort on the part of teachers to develop such, one might well expect that these objectives would be successfully realized. However, there is good reason to believe that this is not the case.

If people possessed a good unterstanding of mathematics would so many of them feel incompetent and express dislike, fear, and yes, even hatred toward the subject? Dr. Nathan Lazar of Ohio State University says, "One in every four people hates mathematics so intensely it can produce mental confusion and even serious emotional imbalance" (2). I question that this condition would exist if these people had a good understanding of basic arithmetic fundamentals. It has been my experience that people of any age or intelligence level are likely to fear and to steer away from that which they do not understand.

Why do so many students after finishing the elementary school abandon the study of mathematics as soon as possible? Some say

* Presented at the meeting of the National Council of Teachers of Mathematics at Dallas, April, 1959.

they "see no sense to it." Many others fail to meet with success. It is generally accepted that arithmetic is the foundation of all mathematics. No one, with the sense that God gave a goose, will continue for very long to build on a foundation that begins to show evidence of not supporting the structure which must ultimately rest on this foundation. He must either strengthen the foundation, or stop building, or be doomed to failure. One has little chance of success in high school and college mathematics unless understanding of basic arithmetic constitutes the foundation for such courses. Many students lacking this basic foundation soon abandon the subject either in high school or in their early college years.

I recently heard the head of a mathematics department in a Big Ten University say that apparently the greatest cause for failures among freshman college mathematics students is lack of thorough grounding in arithmetic rather than inability to understand the new subject matter.

It seems to me that there is considerable evidence that we are not succeeding as well as we should in helping youngsters develop the arithmetical understandings needed to give them the competence and confidence that they should have before leaving the elementary school. This cannot be attributed to the fact that the need for developing understanding has not been brought to the attention of those responsible for instruction. The importance of developing meanings and understandings in arithmetic has been emphasized by leaders in the field for many years. Dr. William Brownell, writing in the *Journal of Educational Research* some 15 years ago, said, "Arithmetic can function in intelligent living only when it is understood" (3).

I believe that most teachers would support the above statement. Why, then, haven't students developed more competency in recognizing, understanding and solving the quantitative aspects of daily life?

I have often wondered why so many students experience an undue amount of difficulty in arithmetic and finally end up expressing fear and dislike for the subject. McSwain and Cooke, in the October, 1958 issue of THE ARITHMETIC TEACHER, state that "The difficulties are in the mind of the pupil rather than in the decimal number system and operations." This answer doesn't help me very much. In fact, it only leads me to other questions, such as: What caused the difficulties to arise in the mind of the pupils? Are there some things that serve as blocks to understanding of arithmetic? If so, what are the blocks and how can they be alleviated?

The Blocks to Understanding

One of our major difficulties in improving instruction may be due to our failure to actually identify the problems that exist for today's children and teachers to understand the concepts of arithmetic. Satisfactory solutions can undoubtedly be worked out once we understand the problems and the causes of the problems.

In no way do I want to infer that I have the long hoped for answer, but I have been interested in exploring the topic and think that I have uncovered a few clues, and perhaps brushed the dust from some others, that may help us unlock the door to better understanding.

For about a year and a half, with the aid of a small grant from the All-University Research Fund at Michigan State University, I was involved in a study in which we attempted to determine: (1) some of the possible blocks to the development of arithmetical understandings and (2) some possible approaches to the removal of these blocks.

The procedure followed in the study involved the securing of basic data from the following sources: (1) educational literature, (2) observations in elementary school classrooms, (3) interviews with elementary school pupils, teachers, and principals, (4) papers written by college students, and (5) a non-computational mathematics test administered in elementary school classrooms.

This study indicates that many pupils in

the later elementary grades show serious weaknesses in such areas as:

(1) The ability to form judgments on the basis of quantitative data, such as, determining whether the data are irrelevant or relevant, and whether the data given are sufficient to solve the problem.

(2) Making reasonable approximations. Many pupils show little ability in estimating reasonable answers or in knowing when and how to round numbers.

(3) Comprehension of quantitative relationships. This fact is also pointed up in several studies reported in the U. S. Office of Education Bulletin, *Analysis of Research in the Teaching of Mathematics 1955 and 1956.* For instance, Melvin Davey in a master's thesis study at Montana State University found that students in grades 6, 7, and 8 are weak in ability to compare the size of numbers and fractions. John Nattkimper, as a result of testing some 280 fifth and sixth graders, concluded in a master's thesis at the University of California, that "very few children responded correctly to items of relationship among processes."

(4) Problem structuring—pupils indicate considerable weaknesses in sensing and analyzing what the problem (or quantitative situation) is about and determining a logical approach to its solution.

There appear to be many blocks to the development of arithmetical understanding and reasoning ability. Some twenty or more possible blocking factors were identified from the data collected in our study at Michigan State. Seldom does it appear that any of these blocks work independently, but rather, that they all contribute to the problem. For our purpose today I shall discuss briefly the eight factors which are most frequently thought of as being blocks to meaning and understanding. The order in which they are listed does not necessarily denote their relative significance as blocks. It does, however, indicate that these blocks are listed here according to their frequency of appearance in the data collected. They are:

(1) Emphasis on memorization and drill at the expense of understanding and thinking. Understanding and quantitative thinking cannot be developed through mere memorization and meaningless drill. When a boy in the 6th grade told me he disliked arithmetic, I asked him why. He answered, "I don't know. I guess I just don't understand what I'm *supposed* to do. It was all right for the first few years. I memorized a lot of things but I guess I have forgotten and now I am having trouble."

(2) Not enough concrete experiences. Before a youngster can become proficient in mathematics he must of necessity be able to deal with abstractions. Too frequently, however, especially in the early grades, we move into the world of arithmetic symbolism before the child is capable of dealing successfully with such abstractions. He needs the experiences of many concrete situations to help him develop meaningful concepts. I still go into many classrooms where the textbook and the workbook are the extent of arithmetic instructional materials to be found either for teacher or pupil use.

(3) Poor understandings on the part of teachers, resulting in poor instruction. This may well be the most important block. This is not to say that teachers aren't interested in doing a good job. In fact, I would guess that most of them are doing the best job that they can. Teachers are probably less well prepared to teach arithmetic than any other area in the elementary school curriculum. Most every study I have seen (and this includes some rather recent ones) on teachers' understanding of arithmetic, indicates that only a very small per cent of practicing teachers possess anything approaching scholarly competence in the subject. This is due partly to the fact that a person can enter many college and university teacher education programs without any high school mathematics and can graduate and be certified to teach without any in col-

lege. It is rather difficult to help someone else understand something which you yourself do not understand.

(4) Fear and dislike of arithmetic. In this respect a student told me recently that "I have never disliked something so much for such a long time. I still remember having to do the multiplication table as punishment. Many a lunch hour I spent doing this. During the 4th and 5th grades I disliked school because of arithmetic." Too often a page of arithmetic seat work is assigned as punishment for a student who misbehaves. This develops a dislike and misconception of what arithmetic is really like. One student recently referred to arithmetic as the "bell subject." When I asked her what she meant, she explained that arithmetic was the "stuff they were given every day in order to keep them quiet between the ringing of the first bell and the ringing of the second bell when school really began." Hence, the name "bell subject." Little wonder this girl looked upon arithmetic as a chore. But it is not only the pupil who expresses this fear and dislike. I find an alarming number of teachers and prospective teachers who show little or no enthusiasm for the subject. How much the teachers' attitude toward a subject area affects the learners' attitude we don't really know. This problem could stand a lot more researching.

(5) Children not physically, socially, or emotionally ready. Even though the developmental pattern of boys and girls is similar in some respects we know it is also different in many other respects for each boy and girl. Why, then, have we been so persistent in arithmetic instruction in trying to get everybody to do the same thing at the same time in the same way?

(6) Lack of understanding of important mathematical terms. Apparently a number of children have difficulty interpreting quantitative situations because they do not know the arithmetical meaning of many terms used to describe the situation. Hazel Forester, in a study to determine the understandings that 4th grade children have of arithmetical terms, considered essential to normal progress at their grade level found that only 15% of the terms were known by 90% to 94% of the children tested. At least part of the difficulty here may be due to the terminology used by educators. Perhaps as suggested by Dr. Morton at last year's convention of the National Council of Teachers of Mathematics, teachers and textbook writers could improve the learners' chance for success if more attention was devoted to improving the language in which quantitative ideas are expressed.

(7) Pupils see little reason for learning arithmetic. We oftentimes do a very poor job of relating anything the children are doing in arithmetic to anything that is realistic in their "here and now" world. Oh yes, teachers do try to help students see a reason for learning. But this effort too often amounts to "you had better learn this, you will need it when you get to high school." This statement may be taken by the child either as a threat—and he may wonder if you can really make him do it—or as something that is too far out in the wild blue yonder to make any difference anyway.

The child may also see no reason for learning arithmetic because he is always told exactly what to do either by the teacher or by the textbook. The rule is given which everybody follows or the problem is structured ready for him to compute the answer. Even the generalizations he should be making for himself may be forced upon him in someone else's wording which he doesn't understand. He seldom has to make any decisions or take any action based upon his previous understandings. No wonder he sees little reason for learning!

(8) Bad reputation of mathematics. Parents and older students may cause young children to expect boredom, difficulty, and even failure in arithmetic. If a student has been constantly told that arithmetic is going to be hard by a parent, older brother, or even sometimes a teacher, he will probably have difficulty until his attitude is changed.

The pupil may have been influenced by Betty Bright who keeps saying "I am so bored with arithmetic, I hope I never hear

the word again. I bet I've just worked a hundred problems all the same kind, and they all have to be done exactly the same way."

Or, he may have been unduly influenced by his buddy John Slow who often had to stay after school because he had not been able to complete the arithmetic assignment every pupil in class had to do before he could go home.

Yes, some students only know arithmetic by its reputation and never by what it really is!

If the above mentioned blocks to pupils' understanding of arithmetic are to be removed, educators as a whole must themselves develop better understanding of both the learner and mathematics as well as continually strive to formulate better methods and techniques of instruction.

Overcoming the Blocks

The following represent some of the most frequently suggested approaches to helping our youth develop more mathematical competence and confidence.

(1) *We should utilize in the classroom more experiences taken from everyday life.* In other words, work more on the things that make sense to pupils. We need to help the child take advantage of the many opportunities in his environment, both within and without the school, to develop meaningful and concrete understandings of numbers. We need to start very early to help the child see the relationship of number symbols to everyday living so that he can move easily, in fact, almost without notice, from the manipulation of concrete objects to the use of abstract arithmetic. Surely one of the best ways of helping children recognize the need and value of studying and understanding arithmetic is to give them a chance to make practical applications of what they are learning to their daily affairs. Bruce Meserve states that "Children start their number concepts by abstracting properties of sets of objects. We must stimulate them to continue thinking of mathematics as a study of properties abstracted from the world in which we live" (4).

(2) *Teachers need to better understand their pupils.* Instruction will undoubtedly continue to improve as teachers gain a better understanding of child development and what this means for effective growth in arithmetic.

The teacher who does the best job of teaching arithmetic is usually the teacher who knows well each pupil in her room. She is well-informed as to the pupil's interests, ability, personal health and adjustment, and background experience.

It is this kind of knowledge that makes it possible for the teacher to differentiate the program so that the study of arithmetic becomes for each pupil an enjoyable, interesting, and profitable experience instead of the frustrating experience it now is for so many children.

(3) *Teachers should place less emphasis on computational speed and more emphasis on the development of mathematical understanding and the ability to reason logically.* Computational ability is still a desirable outcome of studying arithmetic. But I question the desirability of spending valuable instructional time to develop speed in computation, when about all you have to show for the effort is 29 embarrassed and discouraged children who are unable to arrive at a numerical answer as quickly as the one boy or girl in the class who probably least of all in the group needs to experience success. In my opinion, drills or contests to develop computational speed should have gone the way of the old grey mare.

We now have available machines which can do elaborate mathematical computations in a matter of seconds that it would take an expert mathematician hours to do. But we have too few people with the arithmetical know-how to take advantage of these machines.

Arithmetic teachers have too long been overly answer conscious. What we really need to do, if we are going to have good problem solvers, is to help youngsters understand the basic structure of our number system, the "whys" of the various concepts and processes, and how to make practical social applications of their knowledge and understandings.

Scientists and mathematicians have given us good mechanical computing machines. The demand now is for people who can recognize the physical structure or see the sense of the problem situation, and can then express this sense in equation forms through the use of appropriate symbols. Once this is done the person may use his own computational skills to find the answer or he may "feed" the material into the machine to get the answer.

It still remains that the worthwhileness of the computing machine's solution to a problem depends upon the human being's ability properly to structure the problem for computation.

(4) *Arithmetic will be a more enjoyable experience and understanding will be facilitated if the teacher will take advantage of more and better instructional aids.*

There is a wealth of concrete and semi-concrete instructional materials that can be utilized in the classroom to stimulate interest and enthusiasm in arithmetic and to help put meaning and understanding into number concepts and processes.

In the early stages of building concepts the child is greatly helped by the use of manipulative aids. The teacher must realize, of course, that these are, as the name implies, aids to learning and that they will not do the job alone. We can, however, assist the child as he attempts to deal with the many arithmetical concepts, facts, and generalizations by seeing that he has a variety of sensory experiences. He may need to see, to feel or touch, to talk, as well as to hear or listen in order to develop meaning and understanding.

We must do more than we have in the past to help each individual develop clear mental pictures of the truths of arithmetic.

For many years teachers have utilized the blackboard and paper pad as aids for making clear those things which children seemingly have difficulty in understanding. Today's teacher can find, easily available, any number of commercially produced arithmetic A-V aids, devices, and games. Also, many teachers report that fine teaching-learning experiences have occurred when the group has either developed in the classroom or collected from the community materials to aid the student in visualizing mathematics.

We must be willing to explore the educational possibilities of various kinds of instructional materials. A number of experiments with new tools for developing meaning and understanding of mathematics are now being conducted throughout the country, such as those with the Numberaid Abacus, the Abacounter, and the overhead projector. With reference to the latter mentioned aid Dr. Horace Hartsell has this to say: "Probably the greatest innovation for the teaching of arithmetic in the last 10 years has been the introduction of the overhead projector. Its two unique contributions, so vitally needed in the teaching of arithmetic are: (1) a platen, or open face, that allows the teacher to show relationships by super-imposing materials while projecting; and (2) a sizeable work surface for writing and for obtaining a degree of animation needed to express many problem situations" (5).

(5) *Help students to realize that there are a variety of "good" (mathematically correct) ways to solve most problems.* Students need to think, to reason, and to approach each problem from many viewpoints. They need opportunities to pioneer and explore problems on their own, knowing that the teacher recognizes there is more than one way of getting an answer. The job of the teacher is to help the individual child become independent, self-reliant, and confident in working with numbers. I ask you, is there any better way to do this than to encourage the child to use his higher mental processes in exploring realistic problem situations and discovering for himself the rules, processes and generalizations which apply?

(6) *We must do more to help every child experience early and continuing success in arithmetic.* More and more educators are accepting the premise that a child who can learn in other fields can also learn arithmetic. Of course, we know that all children will not experience the same amount of success. It also seems safe to assume that no child is

likely to find the same degree of success in all areas of arithmetic.

It is the teacher's responsibility to determine, as soon as possible, the difficulties a child is having in understanding what numbers are and how to use them. If the difficulty a child is having in the early grades is not soon alleviated, the child can only expect to experience more and more difficulty as he continues his attempts to deal with numbers and number relationships. Instead of progressively building more competence and confidence he is gradually becoming fraught with fright and frustration.

I once heard a student say "I was afraid to ask for help from the teacher because I felt that my knowledge of arithmetic was always behind the rest of the class and I was afraid she'd wonder how I had gotten to the particular grade with so little understanding of the subject." We need more teachers who help children move along successfully in terms of arithmetic meanings and understandings rather than in terms of material to be covered at certain grade levels.

This calls for teachers who can "empathize" with a child who is having difficulty; teachers who encourage children to talk about arithmetic instead of talking so much themselves; teachers who spend considerable time observing children as they work with concrete objects and number symbols, and the way these children approach and solve quantitative situations; teachers who encourage children to diagram or illustrate so as to communicate their mathematical ideas to others; teachers who instill a spirit of inquiry on the part of students and encourage them to think for themselves but at the same time willingly and pleasantly offer encouragement and help when needed; teachers who are competent in arithmetic and do not feel that their own security is challenged when a student comes up with a new, different, and unusual way to solve a problem.

Yes, this is a big order. But I do not believe it is asking too much. In fact, I do not believe we will do much about removing the blocks to arithmetical understanding until we have such a teaching-learning climate established in our elementary school classroom.

In conclusion, I would like to say that if arithmetic is to contribute to intelligent and enriched living (and if it doesn't, it has no place in the curriculum) teachers must help children to see that arithmetic is useful, that it can be exciting and enjoyable, and that *it makes sense*.

References

1. Sueltz, Ben A., Holmes Boynton, and Irene Sauble, "The Measurement of Understanding in Elementary School Mathematics," *The Measurement of Understanding*. Forty-fifth Yearbook of the National Society for the Study of Education, Part I.
2. A Brochure, "You're Helping Eliminate the Fears, Frights, and Frustrations from Arithmetic," distributed by The Ohio State University Development Fund (undated).
3. Brownell, William, "When is Arithmetic Meaningful?" *Journal of Educational Research*, March, 1956.
4. Meserve, Bruce, "A Teacher's View of Mathematics," *School Science and Mathematics*, December, 1956.
5. Hartsell, Horace, "Arithmetic Seen is Arithmetic Learned," *Audiovisual Education*, November, 1958.

EDITOR'S NOTE. Dr. Collier has stated many of the interference factors that affect the learning of arithmetic. He points out that these usually do not exist singly but operate jointly to disturb the learner. He gives some good advice but it is not easy to change many of the school practices that are well established and which are causing the trouble. He points out how important it is for children to have a satisfying early experience with arithmetic and how understanding in the lower elementary school grades will save later frustrations. We can do all of this with most pupils but we are not doing it now in many of our schools.

Why do so many pupils in some schools hate arithmetic while in other schools this is their favorite subject? We should look at the school program, the "climate" of the school, and the teacher. Most often the teacher is the determining factor. The teacher-education colleges could be most helpful if they would provide a reasonable background in arithmetic and mathematics for prospective teachers and if they could and would replace the fear of the subject held by many of their students with at least a sympathetic understanding of the fundamental elements.

Building Algebra Readiness in Grades Seven and Eight*

FRANCIS J. MUELLER
State Teachers College at Towson, Maryland

REVIEW AND EXTENSION of computational processes learned in the elementary grades, mastery of one important new topic —percentage—and the informal introduction of basic geometric concepts are generally accepted as the principal mathematics responsibilities of grades 7 and 8. Less frequently recognized, but of increasing importance, is an added responsibility: building an algebra readiness.

All too often, for those who do recognize this added responsibility, this simply means an earlier—and possibly premature—introduction of topics typically found in ninth grade algebra. It is the contention of this article that a better job of building algebra readiness in grades 7 and 8 can be accomplished by reslanting our teaching of arithmetic in those grades, particularly the review phases, and leaving the specifics of ninth grade algebra to the context of that grade's formal study of the subject. Not only will such a tack afford the pupil better understanding of the vitally important subject of arithmetic in his last formal opportunity to study the subject, but it will create a much firmer footing upon which to build his algebra structure in the future.

The Study of Factors

Of great importance, if the arithmetic program of grades 7 and 8 is to better predispose the pupil for ninth grade algebra, is emphasis on the *factor point of view*. There should be a concerted study of the factors of numbers, numbers as products of other numbers, analysis of multiplication and division in terms of factors and product, and so on. It often escapes the notice of those close to algebra just how great and important a vehicle factors and factoring is in the mechanics of the subject. Throughout the study of algebra we are constantly factoring and expanding, yet in our arithmetic curriculum very little emphasis is placed on this aspect.

It is well to introduce multiplication as repeated addition and division as repeated subtraction in the early grades, for these concepts make greatest sense to the immature minds of youngsters at the age where these operations are first met. But they are not the interpretations that serve us best in multiplication and division of algebraic expressions, so why hold on to them exclusively through the eighth grade? In algebra, the interpretation of multiplication as a product of factors and division as an operation that will yield a missing factor when the product and the other factor are known —the inverse operation of the factor interpretation of multiplication—are the more useful concepts. Yet if we continue to emphasize repeated addition and repeated subtraction as the basic interpretations of multiplication and division in pre-algebra instruction, how can we expect the student to come into algebra with a genuine feeling for the more useful factor interpretations?

Even in addition and subtraction there are factor ideas that have heavy implications for algebra. For instance, we can add 36 and 27 by "factoring out" a 9 from each addend:

$$\left.\begin{aligned} 36 &= 9\times 4 \\ 27 &= 9\times 3 \end{aligned}\right| \quad 36+27=9\times(4+3)=9\times 7=63$$

Why trouble the student with such an involved method? To emphasize the distribu-

* Based on a paper read at the 37th Annual Meeting of the National Council of Teachers of Mathematics, Dallas, Texas, April 4, 1959.

tive law, of course. We already have too many students (and some teachers) who can recognize the distributive law only when it is expressed as

$$a(b+c)=ab+ac$$

because this is about the only way it appears in algebra books.

The time to start talking about the distributive law (and the commutative and associative laws, too, for that matter) is not when we get the student into the very new and strange world of algebra—the time to do it is when he is on the familiar grounds of arithmetic computation.

Is the distributive law important to the student of algebra? That depends upon how algebra is taught. It is not as important to the student when his algebra instruction is patterned along traditional lines as when it is patterned along more modern lines. For instance, the traditional way of showing that $3x+2x=5x$ is often purely by analogy (and an incorrect one): 3 apples+2 apples =5 apples; ergo, $3x+2x=5x$. In a modern context, the student arrives at this result as a consequence of the distributive law:

$$3x+2x=(3)(x)+(2)(x)=(3+2)x=5x$$

Obviously, it would be advantageous for the student to have had already some acquaintance with the distributive law—preferably in arithmetic—before reaching this $3x+2x$ item. Whipping out the distributive law just before we want to call upon it as authority for establishing a new concept, as is usually the case when it is left for the algebra course to introduce, is hardly an effective approach.

In this illustration very simple numbers have been used, 36 and 27. There is no reason why these can't be replaced with more difficult numbers, including decimal and common fractions. In fact, the teacher can "milk" the commutative, associative and distributive laws, as well as the factor methods of multiplication and division, for all the computational review he wants, and in the process not only accomplish his review objective, but also perform a valuable service for the student's future in algebra.

For another item in this vein, let's move over to fractions—addition and subtraction of fractions. Before very long the beginning algebra student is going to be faced with addition and subtraction of algebraic fractions having unlike denominators. On the surface this seems simple enough; the parallel between the arithmetic technique and the algebraic technique is most striking: find a common denominator, preferably the least common denominator, change the fractions, and so on. But how was this student taught to find a least common denominator in arithmetic? More than likely he learned to inspect the denominators; choose the largest; multiply it by 2, by 3, and so on, until he had a denominator that was divisible by all denominators.

Again, we accept the wisdom of teaching this approach at the weaning stages, when fractions are restricted to "ruler" fractions: halves, quarters and eighths, and other simple fractions as thirds and sixths and perhaps twelfths. But imagine finding a least common denominator for a group of algebraic fractions by "choosing the largest," and then multiplying it by 2, by 3, and so on. Parenthetically, how does one tell which is the largest? The one with the more terms? the largest coefficients? or perhaps the highest exponents?

How often in arithmetic courses of study and textbooks do we see least common denominators built up by the "prime factor method," a method that would provide a really useful stepping stone for the student's future work with algebraic fractions? Why can't we do that in grades 7 and 8? Think of all the review work we could accomplish as a consequence of this one variation on the old theme.

Equation-Type Thinking

A second broad area which can be exploited for accomplishing review work in arithmetic, and at the same time better predispose the pupil toward his eventual study of algebra, is that of equations.

In many eighth grade courses of study and textbooks there is a unit on equations. Almost without exception, the treatment is essentially a duplication of what is found in most ninth grade algebra texts. That is, they start with the definition of an equation as a statement that two number expressions are equal, then proceed with illustrations involving a balance scale, and so on.

Since the eighth grade teacher is not tied to the ninth grade algebra text and its inherent continuity (an important item to reckon with when considering deviations in a sequential subject like algebra), why not expose the pupils in the eighth grade—perhaps even the seventh—to the more modern interpretation of equation? Here an equation is looked upon as an open statement that is neither true nor false as it stands, with the variable viewed as a placeholder, or spaceholder, which may be replaced with numerals that will make the statement either true or false.

Modern or not, this approach greatly simplifies the teaching of equations. And even though the pupil should find that his eventual ninth grade algebra instruction is carried out along the usual or traditional lines, at worst, nothing will have been lost. There is serious doubt that the usual work with equations in the eighth grade, which it would displace, is such a great boon to the ninth grade algebra student.

For illustrative purposes, what follows is a rough outline of a possible sequence for introducing equations in this way, the framework of which provides the teacher unlimited opportunities for arithmetic review work.

A start can be made with completion statements, such as the pupil may get on tests in some of his non-arithmetic courses. For instance:

_____ was the President of the United States in 1861.

Insects typically have _____ legs.

One of the branches of our Federal Congress is _____.

Teacher and pupils discuss possible insertions for the blanks; some make the statement true, some make the statement false. Also, some statements have only one possible entry that will make it true; others permit of more than one possibility. In other words, they are talking about domain and solution sets, though use of those words is totally unnecessary. Motivation is not likely to be a problem, at least not at this stage; student attention seldom flags when things that don't look like mathematics are introduced into a mathematics course.

After this, a letter is proposed, such as x, to be used instead of the blank. Gradually, there is a move toward number statements:

$$_____ + 3 = 8$$
$$x + 3 = 8$$
even $$x + 3.62 = 8.07$$

The teacher doesn't worry about formal axioms. He lets the students discover their own devices and techniques.

Then there is a subsequent move into more complex statements. For example:

$$3 \cdot x + x = 20$$
$$2 \cdot x + 3 \cdot x = 10$$

In the beginning with problems of this sort, the students will probably continue to use trial and error methods for finding suitable replacements for the variable—that's all to the good. Eventually, however, some will see the possibility of "lumping" the $2 \cdot x$ and the $3 \cdot x$ to $5x$ (our old friend the distributive law again) and using that in their search for the variable replacement that will make the statement true. The many sorts of possibilities for working in necessary arithmetic review here are obvious.

Other Extensions of Arithmetic

Once this far, there are several useful extensions open to the teacher. Perhaps he will want to move over to statements like $_____ + 3 = 2$, or $x + 3 = 2$. He lets the pupils appreciate the frustration of not having any of his numbers work for that statement; he lets the pupils really sense the

need for a new type of number—the negative.

For good measure along this line, the teacher might uncover statements like: $(__)^2 + 2 = 8$, and see which pupil can come up with the most accurate entry (an interesting workout for decimals). He has the pupils check their results against $\sqrt{6}$, arrived at by the standard algorism, the division approximation method, or from tables.

Still another possibility is to move over to inequalities. These are just another type of true-false statement. The teacher leads the pupils into number statements like:

$$_____ + 6 > 13$$
$$x + 7.36 < 9\tfrac{1}{4}.$$

Here solution sets have many entries or elements. But by combining inequality statements we can sometimes isolate a single number. For instance:

> Billy says John is older than 13.
> Mary says John is under 15.

"What are the whole number solution sets for each of these statements? If both Billy and Mary speak the truth, how old is John?" There are many interesting games, like "20 questions," that can be discussed best in terms of inequalities. It isn't quite linear programming, but then it isn't too far from the basic notion that underlies linear programming, either.

For yet another variation, a start can be made toward understanding *ordered pairs*. The teacher might start with a statement like:

_____ and _____ are brother and sister.

Teacher and pupils discuss possible entries for this statement, where the first blank is restricted to male names and the second to female names. This is followed by a move to number statements that are somewhat similar; perhaps:

$$x + y = 25.$$

What are the possible x-replacements? the y-replacements? What would be the effect if only positive whole number replacements were allowed? If we don't allow x or y replacements to exceed 25? How is the number statement like the brother and sister statement? how different? The teacher has the pupils review graphing and then plot the number-pair possibilities—and in the process gets across the notion that there is a graph for each number statement of this type, and so on. Once more, all sorts of arithmetic drill and review can be accomplished, and at the same time the student's conceptual horizon will be broadened in a way that will be useful to him in his future algebra course.

For a brief final item, let's return to the more commonplace reaches of arithmetic to speak of the complex fraction. Problems and exercises built around this type of expression went out of vogue in the arithmetic curriculum during the "If it isn't realistic, out it goes" putsch. One may readily grant that not many complex fractions occur as the clerk in the supermarket rings up our weekly grocery bill, nor do we find complex fractions important in figuring the gas-mileage on our car, or in computing income tax—but yet, we do know that complex fractions do come up with considerable frequency in scientific formulas and in subsequent mathematics.

As proponents of meaningful mathematics we all strongly favor articulating the subject with reality wherever possible. But who is to say that the demands of mathematical subjects beyond arithmetic represent for the pupil any less a reality than the demands of the market-place? Tailoring junior high school arithmetic to meet only the demands of the consumer in his day-in, day-out transactions is a needless narrowing of the subject. Facility with complex fractions is not only demonstratively useful, but there is no better or more complete way available to the teacher in which to accomplish a thorough review of fractions.

Emphasis on the factor point of view, the modern approach to equations and renewed interest in complex fractions are but three devices that the grade 7 and 8 teacher might utilize to accomplish his arithmetic re-

sponsibilities in those grades, and at the same time better prepare his pupils for their later algebra adventure. Other ways are certain to suggest themselves to the teacher who sees merit in such a double-barreled instructional approach.

EDITOR'S NOTE: It is interesting to note how such almost forgotten items as *least common multiple* are becoming popular again. But this new interest in these topics is not merely one of learning and using an idea just for another topic but one of thinking in a simple mathematical structure and discovering relationships which then become the basis for mathematical insight. Professor Mueller pleads for letting pupils think and discover their own methods of solving equations. They might even arrive at what we call "axioms." This is very different from the typical 9th grade approach and one that probably will yield much more understanding and perhaps less forgetting than a more mechanical method.

The idea of using these extensions of arithmetic as an avenue for added practice of work with numbers may be an interesting new challenge to many teachers. Many will find additional topics to be used in this fashion. Throughout Dr. Mueller's presentation it is apparent that the teacher ought to know some mathematics in order that his teaching be illuminated considerably beyond the realm of direct "presentation-study-master." There will be few haters of mathematics where the teaching is inspired.

Articulation in Arithmetic

It is popularly misconceived and widely mis-practiced to divide one number into another as opposed to dividing one number by another.

More than likely this is an outgrowth of and carry-over from the approach to the method of teaching division which utilizes the following concepts:

1. There are a certain number of times that a divisor can "go into" the dividend, or that

2. there are a certain number of groups, containing the divisor, which can be found in the dividend.

When a division problem is thusly stated $2\overline{)4}$, little difficulty is had with either the solution or the understanding of the functions of the divisor and dividend even if the reasoning used is "two into four," rather than four is divided by two. However, division examples are not always so simply stated; e.g. $4 \div 2$, $2 \div 4$, $4 \div \frac{1}{3}$, $1\frac{1}{3} \div 4$, and $\frac{4}{3}$. And as a group, word problems requiring division are the kinds of examples which complicate the horizon. While most students readily recognize the meaning of the symbols $\div$ and $-$, they most frequently misuse the numbers in examples presented in this manner. Confusion arises as to which one is the dividend and which one is the divisor.

Begin the teaching of simple division examples by:

1. Defining terms
2. Getting pupils accustomed to seeing a division example stated in a variety of ways
3. Insisting that they be able to articulate about the numbers involved and their functions as dividend, divisor, numerator, denominator
4. Establishing equality between the parts and functions of the numbers of one example with that of another stated differently
5. And above all, by establishing the concept that one number is divided by another, so that when the more advanced work of per cent utilizing division (taxation, discount, commission, interest, profit, loss, etc.), ratio, those problems which contain unknown factors and which require algebraic solutions, are encountered an endless amount of confusion will be avoided.

Getting students to be articulate about the above mentioned items may meet with resistance at first, but once the value is accepted, the knowledge assimilated and made use of, the task of learning how and when to apply division is well founded.

This kind of complete understanding helps raise arithmetic above the mechanical level, eliminates the element of chance (that of choosing the correct solution), and, in addition, students learn to recognize and transfer the gained knowledge to other arithmetic problems.

Contributed by
PETER A. STEARN
Leonia, New Jersey, Public Schools

Prime Numbers and Factoring

FREDERICK L. GOODMAN
East Intermediate School, Jackson, Michigan

TO TEACH DISCRIMINATION between prime and composite numbers and to introduce the concept of factoring, a set of baby's blocks makes an excellent teaching aid. Although I have carried out my experiments with eighth graders, I feel the blocks would be useful even in the high school.

The attention given to the unexpected appearance of such toys in a secondary school classroom is part of the value of the method. The visual and manipulative experience provided pin down some rather theoretical ideas. Abstractions become interesting in their own right. The opportunities for humor, as a six-foot adolescent piles the blocks carefully one upon another, are genuine.

The ideas develop simply as you first challenge one of the pupils to arrange seven blocks in any "even" arrangement except that of a single line of seven blocks. By "even" is meant any number of rows or columns producing a level top surface; the same number of blocks in each row or column. After this appears to be impossible, you invite someone to do the same thing with eleven blocks, nineteen blocks, or any prime number of blocks. Finally, suggest someone try the experiment with twelve blocks and then twenty-one blocks, (i.e. any composite number). It is a very short step, after these examples, to the generalization that there are two kinds of numbers.

In stating these first generalizations, it is well to keep in mind that the basic difference in the two kinds of numbers is that one kind can be piled "evenly" or "neatly" in only one way (i.e. a single column or row), while other numbers can be piled in more than just this one way. Care needs to be taken here not to emphasize the idea that a prime number of blocks cannot be piled "evenly," and a composite number can. Pupils must see that the distinction lies in the number of ways the blocks can be arranged.

At this stage, it is well to point out that two blocks belong to the category of things which can be arranged in only one row or column. Note carefully also that one block is, indeed, a special case since we cannot "arrange" or "pile" it at all.

After these two categories are well fixed in the students' minds through further examples, the names *prime* and *composite* can be attached to the appropriate idea. It helps to have the dictionary's definition of "composite" read to crystallize the idea of something made up of parts. This is an important link with the idea of factoriug.

The students should be able to distinguish between primes and composites without referring to the blocks before proceeding to factorization, but for this introduction of factoring the blocks should be used again.

Challenge four students to "build a model" of the number twelve, each in a different way. Imply from the outset that they will have to use some imagination (for one of the students will have to arrange the blocks in two groups, each of which contains two columns of three blocks, or three columns of two blocks, in order to get four different models). Tell them that you will not consider six columns of two blocks to be different than two columns of six blocks. The arrangement into more than one group might have to be suggested specifically by the teacher.

As each of the pupils completes his model,

list on the blackboard the number of blocks across, the number high, and the number of rows counting front-to-back. For example, if the first student chooses four columns, each with three blocks in it, and places them so that the class can see the broad side of the arrangement, list on the board 4·3·1. Ask the class to explain the origin of the numbers you have written and be sure everybody in the room sees the "4 across, 3 high, and 1 from-front-to-back" idea. You should end up with the lists, 4·3·1, 6·2·1, 12·1, and 2·2·3·1. In the "12·1," I dismissed the idea that I wrote only one "1" rather lightly. even though I might have logically written 12·1·1. In the "2·2·3·1," one of the numbers will refer to the number of groups made.

If the students are not familiar with a dot representing a multiplication sign, I would write the lists 4×3×1, etc. It is important that the realization eventually comes to each pupil that all of the lists contain numbers which, if multiplied together, will equal 12. If it is not clear after only the example of twelve blocks is used, I would suggest that the students be kept experimenting with different numbers of blocks until they ask about, or tell you about, this relationship themselves.

It seems quite crucial, however, that they see these factors as *parts* of the number being factored, rather than numbers which they merely string together with the idea of getting a combination which will multiply out to the right number. The concept of taking a number apart and putting it back together is important in mathematics. Many students never seem to sense this relationship fully.

Actually, the term "factor" may be introduced at any time. More important is the idea that some numbers and their factors cannot be illustrated well with the blocks. 32=2·2·2·2·2 gets to be a little far-fetched for instance. This is a good opportunity to suggest that mechanical analogies are only introductions to mathematics, and that many mathematical ideas cannot be represented in any simple physical sense.

A summary with regard to prime numbers and composite numbers is worthwhile once the word *factor* is added to the class's vocabulary. Lead them to state that seven blocks can only be "factored" one way rather than "piled" one way. Furthermore, the factors of primes always seem to be the number itself and the number 1. I found that this statement could be drawn from the better students quite easily, and that the rest of the class was in full agreement.

After numbers can be factored and identified by name using blocks, it is well to lead them quickly away from that unnecessary crutch. In most cases the pupils themselves are the first to claim the blocks have become a waste of time.

Depending on the time you wish to spend and the objectives of your program, the topic can be dropped here or carried on to consider odd and even numbers, the notion of greatest common factor, the discovery of the least common multiple, and applications to fractions. I used from one to two weeks covering all these topics, with the time depending on the ability of the class. It may be interesting to note that this approach was used with one class containing children with I.Q.'s hovering around 70. The results were as stimulating, in their own way, as were those obtained in classes preparing for algebra.

EDITOR'S NOTE. Yes, Mr. Goodman uses blocks to illustrate prime and composite numbers but he doesn't just illustrate, he has pupils discover and tell him and the class what generalizations they have discovered. And he recognizes that blocks should be discarded as soon as they have served their purpose. Basically he is interested in the make-up or structure of a number and how this can be used to enhance the understanding of arithmetic. L.C.M. is but one outcome from a study of factors. G.C.D. also has a role in working with numbers.

Stocks as Investments

THOMAS J. GANLEY
Andover Junior High School, Andover, Mass.

"My stock went up $7\frac{1}{4}$ in the last five days." An eighth grade boy had just finished reporting on a company listed on the New York Stock Exchange that he had been keeping a record of as part of his class's assignment on a unit about "Investments and Savings."

I began the study of Stocks and Bonds with my eighth grade classes with a vocabulary lesson concerned with the various words and terms associated with the Stock Market. Such words as dividend, coupon, bond, brokerage, corporation, below par, market price, investment, stockholder, common stock, preferred stock, quotation, prospectus, net change, bid, stock broker, open, high, and close were some of the many words and terms that the pupils were to look up the meanings of and be able to use in a sentence or give an illustration of. Their homework assignment was to bring in the financial section of a newspaper for the next day.

With newspapers in hand, and opened to the financial sections, we discussed the headings at the top of the many listings of stocks and put some of our definitions to use. The pupils learned how to interpret what these headings meant and what the abbreviations were along with the symbols that are sometimes attached to some of the listings. Articles and forecasts of financial trading, dividend notices, and bonds advertised for sale were pointed out to them and discussed.

I then assigned them to select a stock listed on the New York Stock Exchange from the financial section of the newspapers and to make a record or a graph of the day to day trading and progress of "their stock," showing its net changes daily and at the end of five days to report their findings.

I had scheduled three films to go along with our study of the Stock Market, each one to be viewed a week apart. The first, entitled "Your Share in Tomorrow," explained the origin of the Stock Market, Wall Street today, bidding, buying and selling, and what the future has in store for America as regards finances and progress in industry. A discussion and a quiz followed each movie.

At this particular point the pupils' interest and enthusiasm were very high and it wasn't long after that the following material began to appear on my desk without my having assigned them any further research: "Monthly Stocks Digests," "Standard Listed Stock Reports," quarterly and annual reports from companies listed on the stock exchange, prospectuses, various other articles and pamphlets, and even a manual explaining Stock Exchange Procedure.

The pupils were eager to discuss their literature with their own classmates and so I allowed time at the beginning of each period for explanations and discussions of this material.

About this time an article appeared in *The Saturday Evening Post* in the February 7, 1959 issue entitled—"What's Behind the Stock Market Turmoil?" written by Harold H. Martin. I brought it into class intending to read some excerpts from it, but was surprised to learn that my pupils were way ahead of me. Not only had they purchased and read the article, but upon questioning them about the article they understood it.

Our bulletin board in the classroom began to appear somewhat like a broker's office in that much from the aforementioned articles was posted on the board along with a daily newspaper displaying the financial section, which was changed every day.

Assignments from the pupils' text were made in the arithmetics of buying and selling shares in companies, finding rate of income on stocks, profits and losses, buying bonds, and current yield. I stressed at this point the tie-in of their previous study of "Type II" per cent.

The second week of our study of the Stock Market, we viewed our second film, "Working Dollars," a story of the monthly investment plan and the necessity of investigating before investing in a company.

I wound up our study of "Stocks As Investments" by a third film called "What Makes Us Tick?" This film differed from the previous two in that it discussed and explained that part of the stock market from the owner of a company's viewpoint. How he gets his company or business approved for listing on the Stock Exchange and, when once approved, how he enlists the aid of Mr. and Mrs. America by selling shares in his company and later the distribution of profit as dividends to the shareholders. Finally, the reading of their own reports, review, and a written examination completed our study. The "mystery" of the Stock Market had been solved, and they were left with the satisfaction that they were now better able to comprehend the market, that you need not be a person of great wealth to be a shareholder, every person should try to save a part of his earnings, and that ownership in a company is also putting your faith in the progress of our nation.

As for me? I learned something too. Students today, even of Junior High age, are interested in the future of America, their own future.

My satisfaction? A rewarding experience! Observing my pupils' amazement and constant interest that never faltered throughout the unit. The countless questions that arose in their minds displayed their eagerness to learn more about something that they supposed was to be left for a more adult person.

EDITOR'S NOTE. In recent years and especially through "mutual funds" many people including school teachers have become shareholders in the great American adventure of investing. In the larger urban areas and particularly in the Eastern section of the country one finds many holders of stocks listed on the public exchanges. Inasmuch as money plays so important a role in our society, why not teach pupils something about wise buying, investment, and thrift? Many people line up at the savings banks semi-annually to have the 3 or 3½% interest recorded in the little book and then on the way home pass up 12% saving at the food market. And to accentuate the difference, the 3% is on an annual basis whereas the 12% may be repeated several times per year. Is it the province of mathematics to teach these things or should they be left to casual association outside of school and perhaps never learned?

Mr. Ganley reports a very high interest among the pupils and no doubt his own competence and enthusiasm were factors in this. There are many values other than just information and computation that can be gained from a topic such as investment. It is usually both illuminating and shocking when a stockholder first learns the large amounts of taxes paid by corporations and how this must be deducted before a net profit is secured. The factors of risk versus safety and yield versus security and many other concomitants are inherent when mathematics is applied to business and industry.

Announcement

THE ARITHMETIC TEACHER will henceforth be issued eight times per year during the months October through May. The Board of Directors voted this change at the annual meeting held in Dallas last April. Thus the two Journals THE MATHEMATICS TEACHER and THE ARITHMETIC TEACHER will have the same number of issues annually. In six years THE ARITHMETIC TEACHER has grown from zero to ten thousand circulation. The National Council is happy to increase the number of issues from six to eight in token of the fine reception accorded to THE ARITHMETIC TEACHER.

An Adventure in Topology—Grade 5

JEAN C. CLANCY
Scarsdale Public Schools, Scarsdale, N. Y.

TO TAKE AN ADVENTURE implies the unknown, and that is just what a group of fifth grade children did recently in mathematics. Before such a trip could be taken, the consent of all had to be obtained. In this case the pupils had a desire to learn and the teacher who acted as the guide, wished to stir the imaginations of her pupils.

In taking inventory she had to ask herself some questions. Was the present math program adequate for all pupils? Was computation the most important phase of arithmetic to be taught? Were the pupils really motivated to think in mathematical terms?

To answer the first question she would have to do some experimentation to discover just how adequate the program was, for she realized that although many pupils could handle advanced work easily, something more was needed to broaden their experience in mathematics.

As for computation, this was a skill worth developing, but not to the exclusion of others. If mathematics was completely manipulation of numbers, the electronic machines could "take over." A child's arithmetic skill and such a machine have one thing in common—both are in need of repair from time to time—but unlike the machine, a child can develop the ability to reason and use logic.

To motivate children to think in mathematical terms is a real challenge, for of all the science mathematics breeds a motivation that is different from that required in other academic areas. The "self-felt" needs aren't always seen in mathematics, and despite present day concern for greater competence in math, this science has *always* been the most challenging group of ideas to teach students.

If the necessary skills beyond computing aren't apparent to all teachers—whatever level—how can "needs" be communicated to students? To meet the challenge of motivation, she felt an obligation to approach math in a way that was new to the pupils and herself; to begin looking for patterns of thought that have their basis in mathematics; to be able to generalize, work with theory and then make some applications that would be appropriate to the classroom situation.

The ideas took form in the beginning of the year when the fifth grade class was discussing length and width and then began to ask questions about other dimensions, and the meaning of our three dimensional world. When paper was represented as 2-D, the class was asked if they had ever seen one-sided paper. After the initial laughter and questions had subsided, they were asked to think about this and share their thoughts with classmates the next morning. Their thinking ranged from: "paper stapled to the bulletin board so only one side showed,' to "a picture of a piece of paper because only one side could be photographed." Certainly there was thinking, and above all a genuine curiosity about one-sided paper.

When the teacher cut a long narrow strip of paper and pasted the ends together, in the form of a headband, the class told her that "it has two sides because we can see two sides—inside and outside." Then she took another strip of paper—longer than it was wide—and gave the paper a twist before joining the two ends together (see Figures 1 and 2). She told them this was called the Mobius Band named for a German mathematician who studied it about 100 years ago. To help them discover why this was one

FIG. 1

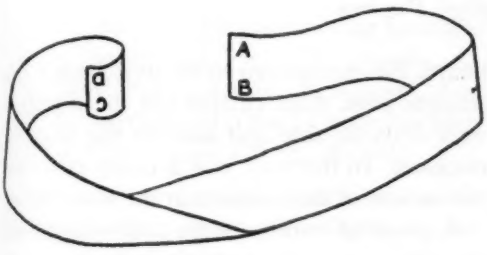

FIG. 2

sided paper, she had them run their fingers down the middle of the length of the strip, following the twist carefully. After one trip around the band, they found that they had returned to the original point of departure, without having taken their fingers from the paper. No edges of paper had been crossed.

When trying the same procedure with the first named ring (in the form of a headband) they discovered that there was no way to get from the inside circle to the outside circle without crossing an edge. Therefore the conclusion was that the Mobius surface was indeed an example of one-sided paper whereas the "ring" was the usual two-sided type.

Interesting to fifth graders? Yes, especially when discussion led to the three dimensional world we live in and applying what they already knew about dimensions, to our world.

The motivation for more thinking was in the new found surface before them—the Mobius Band. Yet why learn about something that had no apparent significance to them in their three dimensional world? With more thinking and reasoning taking place, some in the group ventured a "hunch" that it must have something to do with outer space. This proved to be a good idea for it was pointed out that in four-dimensional space, closed surfaces such as Mobius can exist. Mathematicians and scientists believe that it isn't at all impossible that astronomical space is closed in on itself and twisted like the Mobius Band. Amazing!

Although this would be the closest they could ever come to a Mobius Band, still it would be fun to experiment with it to see if it had any unusual properties. By cutting lengthwise through the middle, everyone was prepared to find that the band would be cut into two pieces. But somehow the unexpected happened, for instead of having two pieces, it still had one piece, but twisted twice instead of once. And worst of all it was now just an ordinary two-sided band! The fifth graders decided that this magic was all right for demonstration, but were just as glad that outer space in the form of a giant Mobius surface was free from prying scissors!

When someone recalled that you, the teacher, had mentioned four-dimensional space and "what did you mean by that because here we only have three dimensions?" it demanded all your resources to explain to ten year olds that *time* was the fourth dimension.

It didn't seem too mysterious to understand when it was explained that events that happen around us not only include distance in space but also the *time* that they happened to be completely descriptive.

Now that the measurement of space was in the thoughts of all, another word could be added to the growing ten's vocabulary—along with dimension, Mobius, surface, length, width, height. This word was Topology, the branch of geometry which means the study of locations (without reference to measurements of lengths or angles in space).

"If this is a part of geometry—the subject that my brother is studying in high school—couldn't I learn something about it too, and have sort of an adventure like we did with the Mobius Band?"

"Yes, as a matter of fact you could"—and all of us did have an adventure in geometry that led us to the very nature of things around us.

(*Concluded on page 256*)

Uses of Styrofoam in Arithmetic

ADRIEN L. HESS AND LILLIAN KEARBY
Montana State College, Bozeman

ONE OF THE FIRST RULES of teaching is to catch and hold interest, for without interest much of the teaching effort is wasted. If a learning situation can be made a source of fun, something that will catch and hold interest, then one can expect a high degree of permanence of learning.

Modern elementary school teachers are constantly searching for means and materials for developing concepts by concrete presentation. It has been found that younger children learn more readily if the material presented is in the form of something they can hold and see. Ideally, these materials should possess three characteristics; they should be economical in point of cost and time, they should be neat and attractive, and they should fill a definite teaching need.

Styrofoam is very useful in demonstrating the principles of arithmetic in a concrete way. Cutouts and devices made out of styrofoam make excellent materials for many teaching situations. Such cutouts and devices may be made very attractive and since they adhere readily to the flannel board they can be used in many ways. They have an advantage over flannel cutouts in that they are easier for little children to manipulate. Children may hold the objects being used and often may put the pieces in place on the flannel board. This serves not only to hold interest better but also increases learning by providing for active class participation.

Styrofoam comes in white, green, blue and red sheets. These usually are about two inches thick. If other colors are desired, a spray paint can be used on the foam.

In order to cut an object out of styrofoam it is best to use a pattern as it is a bit difficult to cut free-hand. To make numbers, use stencils to make a pattern. It has been found that the patterns are more usable if made out of tag board or some other light weight cardboard. Pin the pattern to the styrofoam with straight pins. After cutting out the number turn it on its side and slice to the desired thickness. In this way, one actually cuts out two to four of the numbers at the same time.

A piece of bare wire through which an electric current is passing makes an excellent cutting device. The wire must be kept taut and a rheostat or similar device must be used to cut down the current. The heated wire cuts the styrofoam readily and gives a smooth appearance to the cutouts. There are numerous ways in which to cut out the centers of the 4's, 6's, 8's and 9's. Use either a razor blade, sharp knife or even a sharp point from a wood-burning set.

Elmer's glue may be used to put two pieces of styrofoam together; however, care must be exercised in not using it to excess as it will melt the styrofoam.

If one has some artistic taste there is no limit to the interesting and attractive devices that can be made. By using different colors one can make beautiful devices, such as fractional pies, and a score of other helps such as number combinations. Another clever device in the form of a clock can be made for teaching time. The numbers and hands are made of a different color than the background. Use a small hat pin in the center to hold the hands in place and in this way they can be changed repeatedly to the desired time. This makes a much more attractive device than those made of cardboard or other such materials.

EDITOR'S NOTE. Styrofoam, sponge rubber, and other new synthetics are available for a multitude of devices to enhance the representation of concepts in arithmetic. But as the authors point out, these materials are not intended for "busywork" but for showing an important concept or principle. Many school supply catalogues and most hobby shops carry these new materials and many of them are not expensive.

Reviews of Books and Materials

Edited by E. GLENADINE GIBB

Teaching Arithmetic for Understanding (and student handbook), John L. Marks, C. Richard Purdy, Lucien B. Kinney. New York: McGraw-Hill Book Company, Inc., 1958. Cloth, xiv+429 pp., $6.00.

Many of the criticisms stated here apply equally well to most other books on the teaching of arithmetic. On some of these points the present book is better than most.

I find no clear statement of the nature of mathematics as the science of deductive reasoning, nor of mathematics as an art and as a language, nor of the role of mathematics in the natural and social sciences. I find little mention of mathematicians, nor any hint that mathematics is created by human beings. A reference to Bell, *Men of Mathematics*, and to some book on the history of mathematics, would help. Every teacher should know the book of Kline, *Mathematics in Western Culture*. There is no adequate reference to career information and the needs for mathematical training in various vocations, nor to the manpower needs of our society, such as the Steelman report, *Science and Public Policy*. The outline of the aims of arithmetic study on page 12–14 is unsystematic and confusing, and is hardly a model of clear thinking for the prospective teacher. There is no mention of the impact of high-speed computers on our society. There is no mention of elementary number theory either in the text or the teachers' manual. Such a book as Ore, *Number Theory and its History*, is an invaluable source of enrichment material. How can a teacher or a teacher of teachers make arithmetic interesting if he doesn't know anything interesting about numbers? There is no reference to Swain, *Understanding Arithmetic*, which is the best book I know on the mathematical content of arithmetic.

The authors do not mention the fundamental work of Piaget on the development of mathematical concepts in children. A teacher would find helpful a summary of the research on learning and reinforcement and its application to the scheduling of drill.

The authors recommend, essentially, the grade placement of topics suggested by the Committee of Seven. Already 30 years ago Brownell published a devastating analysis of this research. Yet the results are used as though they provide reliable evidence for the design of the curriculum. In England, Sweden, and other countries, children learn in 4 years what our pupils learn in 6. The report of Buswell indicates that British children do learn this curriculum effectively. The concept of "social uses" of arithmetic is narrowly limited to routine computations and ignores the applications to the natural and social sciences.

There is a good discussion of the meaning of the decimal system of notation and alternate bases. It is not made clear that numerals are names of numbers, and that the content of arithmetic is the properties of numbers. The notation must be understood, but is incidental, as a tool of communication, to the main business of teaching arithmetic.

The authors lay a welcome stress on the laws of arithmetic, and discuss explicitly the commutative and distributive laws. The associative law appears disguised as the law of "compensation." These generalizations are well stated in words, but the authors apparently feel that the statements in symbols are too difficult not only for teachers, but also for teachers of teachers. Average 6th graders learn with delight that if a, b, and c are any numbers, then

$$a\cdot(b+c)=(a\cdot b)+(a\cdot c),$$

and that this is the explanation of the usual process for computing

$$3\times 21=3\times(20+1)=(3\times 20)+(3\times 1).$$

Like all educators, the authors advocate adjustment for individual differences, yet do not, apparently, apply this philosophy in their own teaching. There is nothing in the text adapted to the capacities of a student who has had 9th grade algebra, let alone any college mathematics. I can think of no policy more likely to discourage a well-prepared student from going into elementary school teaching.

PAUL C. ROSENBLOOM

The Growth of Mathematical Ideas, Grades K-12, Twenty-Fourth Yearbook, NCTM. Washington, D. C.: National Council of Teachers of Mathematics, 1959. Cloth, x–507 pp., $5.00 ($4.00 to Council members).

For the teacher who is looking for a quick course in *Teaching of Arithmetic Made Easy* the writer would not recommend this book. For the teacher seriously wishing to improve the teaching of arithmetic, however, and willing to spend some time studying, it has much to offer.

A thorough perusal of the book will enable those engaged in teaching children at any level to have a comprehensive view of what consistutes a good K-12 mathematics program, and ways and means of presenting it to children.

The authors have approached their objectives with an apparent concern for the teacher of meager mathematical background. They have taken care to show how the foundation for quantitative thinking must be carefully laid through many meaningful experiences in the very beginning of elementary school. They pursue their objectives throughout the book by showing how these foundations are refined and expanded, and finally they show that by systematic progression from these first simple, meaningful experiences, facility with abstract quantitative thinking is eventually achieved.

Happily the authors give as much importance to one level of instruction in the K-12 sequence as to any other level. The kindergarten teacher will surely gain the impression that number experiences at that level contribute vitally to the pupils' total mathematical insight and achievement. It is made clear that no mathematical idea rightly presented, developed and applied, ever becomes useless cargo, but rather remains an indispensable part of the pupils cumulative knowledge of mathematics.

Of especial help to inexperienced teachers or those not well grounded in mathematics is the discussion of the major understandings, concepts and insights as they are introduced in the K-12 program. One example of such is to be found under *Number-Numeral* (p. 332). Here the usually-so-called *answer* is presented as *another name*. Even the young child, although he cannot express it, senses that the *name* of anything means the essential nature or character of the thing (Daddy, Mother, family, pet, toy). It is not difficult for him to comprehend that if 37 is another name for 28+9, then 37 must have the character or nature of 28+9; or to understand that 37 is another name for

$$28+(2+7)=(28+2)+7=30+7=37,$$

because it has the *character* of 37. Another most timely discussion is concerned with *The Concept of Set and Cardinal Number* (pp. 10–17, 65–67, 71, 88). If the elementary teacher has here-to-fore felt that the idea of *set* was consigned to higher mathematics this discussion is enlightening. Here, the idea of sets and their possibilities for developing insights and understandings in working with cardinal numbers is discussed in some detail. The elementary teacher should find the content of these pages very helpful, and should no longer avoid the use of sets for the reason that they are not understood.

The topic of place value has been so tossed about in the past few years that many teachers have become confused about its proper role in mathematics. In this book place value is shown to be *not a property of*

number (as one gathers from some textbooks and professional readings), but, rather, an operational principle of the decimal number system upon which rests the rationale of the four fundamental operations. It is shown that this operational principle is used not only with natural numbers, but also with decimal and common fractions. How this operational principle works is explained in detail.

Although this book outlines certain concepts, understandings, generalizations, and processes which must be acquired sequentially from year to year, it does not specifically limit certain of these to a particular grade level. It places emphasis on quantification rather than on computation, but repeatedly points out that developing insights and understandings should lead to the acquisition of computational skills.

Throughout this book references and suggestions for presenting, developing, extending and applying mathematical ideas are set forth.

One particularly interesting section is *Activities for Elementary School* beginning on page 232, which suggests introducing activities dealing in an informal way with the ideas of probability as early as kindergarten. The section goes on to suggest activities for extending and expanding these ideas at all elementary grade levels, and shows how such work sets the stage for graphing and coordinate geometry later.

The authors recognize that no one book can encompass all of the continuing ideas of mathematics, nor point out entirely their recurrence throughout Grades K to 12, much less present in completeness classroom procedures for extending them throughout the pupils' K-12 experience. Nevertheless, this writer believes that any teacher who studies this 24th Yearbook of NCTM, will surely enrich his own knowledge of mathematics, and gain many ideas for motivating, and developing mathematical ideas in the classroom.

RUTH H. TUTTLE

Film Review

"Donald in Mathemagic Land"

This is a 26-minute technicolor film created by the Disney Studios and which is now being released to theatres throughout the country. "Its purpose is to strip that fearsome word 'mathematics' of its mystery and make it a friendly household word for kids and adults alike." Dr. Heinz Haber provided technical counsel in the preparation of the film.

The picture opens with Donald Duck exploring Mathemagic Land—a fantasy land composed of such things as square root trees and a stream running with numbers. He is startled by an offstage voice—the Spirit of Adventure. The Spirit tells Donald that he is about to enter one of the greatest adventures of all, Mathematics. The Voice then transports Donald to Ancient Greece. Here Donald meets ghostly mathematicians of the past. He learns about Pythagoras and how he discovered mathematics in music. Donald joins in an Ancient Greek Jam Session.

Donald now learns that the Ancient Greeks discovered mathematics in art, architecture and nature. Here, to music, we see the Golden Rectangle, an Ancient Greek formula for beauty. Donald follows it and sees it fit Greek architecture and more modern edifices. He finds that this rectangle even fits the human body. The Pentagram, a star made from golden proportions, fits into many things in nature . . . starfish, shells, flowers, ferns, etc.

Next Donald gets mixed up in a chess game. He becomes Alice in Wonderland and is chased by giant chess pieces. He grows large and finds himself beside a normal chess board. He now finds mathematics in chess as he watches a game played out in speeded action. The Spirit then shows Donald that there is mathematics in many games . . . football, basketball, baseball, etc. Surprisingly enough in billiards, too. An expert shows how it works in billiards with a

display of fantastic shots. This is all accomplished by clear thinking, Donald learns. Donald tries this method and is successful.

The Spirit then shows Donald that in order to think clearly one must clean out the antiquated ideas. Donald now gets a clear picture of mathematical thinking. He discovers infinity. Donald discovers simple geometrical figures—the circle and the triangle—have been the basis for many great inventions—telescope, airplane, etc. . . . that it is this type of thinking that will open the doors to the future.

We end the picture on Galileo's quotation "MATHEMATICS IS THE ALPHABET IN WHICH GOD WROTE THE UNIVERSE."

BEN A. SUELTZ

The National Council of Teachers of Mathematics

Operating Budget

1959–60

RECEIPTS

Memberships (19,549 @ $5.00)	$97,745.00	
Memberships (Students: 1,400 @ $1.50)	2,100.00	
MSJ Subscriptions (65,000 @ $0.30)	19,500.00	
Membership (for second journal: 4,350 @ $3.00)	13,050.00	
Subscriptions (both journals: 3,900 @ $7.00)	27,300.00	
Subscriptions (both journals: 3,900 @ $6.75)	26,325.00	
Advertising in journals	7,800.00	
Interest on U. S. Treasury Bonds	375.00	
TOTAL		$194,195.00

EXPENDITURES

Washington Office		
Executive Secretary		
Salary	$10,825.00	
Special benefits	1,200.00	
Professional Assistant		
Salary	7,500.00	
Special benefits	180.00	
Editorial Assistant		
Salary	6,000.00	
Special benefits	170.00	
Secretarial and clerical help		
Salaries	50,000.00	
Special benefits	1,850.00	

General		
Office expense	14,000.00	
Travel	3,000.00	
TOTAL		$ 94,725.00
President's Office		
Office expense	$ 700.00	
Travel	2,000.00	
President's Fund	700.00	
Secretary	1,820.00	
TOTAL		$ 5,220.00
Vice-Presidents		
Postage and secretarial help	$ 2,000.00	
TOTAL		$ 2,000.00
MATHEMATICS TEACHER		
Printing and mailing	$36,000.00	
Office expenses and secretary	4,500.00	
Travel, editor	500.00	
TOTAL		$ 41,000.00
ARITHMETIC TEACHER		
Printing and mailing	$18,000.00	
Office expenses and secretary	2,000.00	
Travel, editor	500.00	
TOTAL		$ 20,500.00
MATHEMATICS STUDENT JOURNAL		
Printing and mailing	$12,000.00	
Office expenses	1,000.00	
Travel, editor	500.00	
TOTAL		$ 13,500.00
Other		
Committee work	$5,000.00	
Travel, directors, vice-presidents and recording secretary	5,000.00	
Convention program expenses	1,500.00	
Audit	750.00	
Special Projects Fund	5,000.00	
TOTAL		$ 17,250.00
TOTAL EXPENDITURES		$194,195.00

Subtraction from Left to Right

The conventional method of subtraction presents certain difficulties connected with the "borrowing" process. Either corrections in the minuend must be made on paper or a burden is thrown on the memory. Of course, the farther over one reaches for the "borrowed" figure, the more confusing the situation may become. The student may err through failing to make compensatory changes or through making changes wrongly.

So long as one proceeds from right to left no remedy is possible. But as soon as we approach the problem from the other end, that is, from left to right, from the large units to the smaller ones, a new possibility presents itself. Instead of "borrowing" we can *give*, and with this distinct advantage: all adjustments are immediate and final. Also, once the necessary gifts are made, no columns exist in which the subtrahend is greater than the minuend. What appears to be 2−5 is really 12−5.

How, then, do we make the necessary gifts? By following this rule: as we proceed each difference must be diminished by 1 *if* a set or column in which the subtrahend is larger occurs before one in which the minuend is larger. (In diminishing 0 by 1 the answer is 9.)

For example:

$$\begin{array}{r} 7627320 \\ -4284328 \\ \hline \end{array}$$

Starting with the millions we have 7−4 and get 3, which is correct because in the next set the minuend 6 is larger than the subtrahend 2. However in doing this next set, the resulting 4 must be reduced by 1 to 3 because in the set following the 2 on top is obviously smaller than the 8 on the bottom. Continuing with 12−8 equals 4, the 4 is *not* reduced because in the next set the top number 7 is larger than the bottom number 4. But when we take 4 from 7 we must change the resulting 3 to 2 because, looking beyond the next two sets which both have equal minuends and subtrahends, we come to a set in which the minuend (0) is smaller than the subtrahend (8). We continue with 3−3 equals 0, but this 0 must be "reduced" by 1 to 9 for the same cause as above (where we corrected the 3 to 2). Also, the 0 resulting from 2−2 must become 9, again for the same reason. In the last or units set, 8 from 10 is 2, which holds.

With a little practice long subtraction examples may be worked from left to right almost at a glance.

Contributed by
DAVID ERLICH
Fulton Elementary School, Philadelphia

Virgil S. Mallory

Dr. Virgil S. Mallory died at his summer home at Sharon Springs, New York on September 30, 1959. Dr. Mallory long held the position of Professor of Mathematics at Montclair New Jersey State Teachers College. He was very well known and respected for his contributions to the field of mathematics education not only through his teaching but also through his authorship of many textbooks and for his services to the New Jersey Association of Mathematics Teachers and to the National Council of Teachers of Mathematics. Since his retirement from Montclair, Dr. Mallory continued writing and speaking. His loss will be felt by the many people who had learned to know him as one of the leaders in the teaching of mathematics during the past quarter of a century.